Pre-Midterm Review Topics

- Be able to put complex numbers in exponential form
- Know how to calculate roots and powers of complex numbers
- Know how to compute matrix-vector and matrix-matrix products
- Reduction of matrices to echelon form
- Identify matrices in echelon form
- Identify pivot and free variables from a matrix in echelon form
- Know how to determine existence and types of solutions
- Reduction to row canonical form
- Calculation of matrix inverse
- Know how to use matrix inverse to solve a system of equations
- Understand linear dependence and independence (also in relation to linear combinations)
- Know how to find a basis for a span of a given set of vectors
- Find a basis for rowsp(A) and colsp(A)
- Know how to determine the rank of a matrix
- Know how to determine the dim of a subspace spanned by a set of vectors
- Understand Rank-nullity theorem and how to deduce the dimension of the kernel of a matrix
- Know dim(\( \mathbb{R}^n \)) = \( n \)
- Understand concepts of spans and subspaces
- Know how to determine if \( \text{span}(u_1, \ldots, u_m) = \mathbb{R}^n \)
- Know conditions for existence of a matrix inverse

Post-Midterm Review Topics

- Computing eigenvalues
- Computing eigenvectors
- Understand algebraic and geometric multiplicity of eigenvalues and relationship to diagonalizability of matrices
- Determining invertibility of a matrix
- Determining if a matrix is diagonalizable
- Diagonalization of a matrix
- Computing determinants
• \( \det(A) = \lambda_1 \lambda_2 \ldots \lambda_n \)

• Computing orthogonal complement of a vector subspace

• Computing dimension of an orthogonal complement of a subspace

• Computing distance between two vectors

• Computing inner dot product for two vectors

• Computing projection of vector onto a subspace

• Finding coefficients in expansion of a vector as a linear combination of an orthogonal basis

• Determine orthogonality of a set of vectors

**Additional Review Problems**

Determine whether the following statements are true or false

1. Two row equivalent matrices have the same rank
2. There exists a \( 3 \times 2 \) matrix with rank 3.
3. A homogeneous linear equation always has a solution
4. If a \( 3 \times 3 \) matrix \( A \) has a zero row, then rank \( A = 2 \).
5. If \( v \in \mathbb{R}^n \), then \( -v \) is in the span\{v\}
6. Let \( u \in \mathbb{R}^n \) and \( v \in \mathbb{R}^n \); then span\{u, u - v\} contains \( v \).
7. If a \( 6 \times 4 \) matrix \( A \) has linearly independent columns, then the echelon form of \( A \) contains two zero rows.
8. There exist a \( 3 \times 5 \) matrix whose column space has dimension 4.
9. If a square matrix \( A \) has two identical columns, then \( \det(A) = 0 \).
10. If \( \lambda = 0 \) is an eigenvalue of the square matrix \( A \), then \( A \) is invertible.
11. Every invertible matrix is diagonalizable.
12. Every diagonalizable matrix is invertible.
13. The set of all solutions of a system of homogeneous equation with \( m \) equations and \( n \) unknowns is a subspace in \( \mathbb{R}^n \).
14. The set of all linear combinations of columns of an \( m \times n \) matrix is a subspace in \( \mathbb{R}^n \).
15. The columns of an \( n \times n \) matrix \( A \) form a basis for \( \text{colsp}(A) \).
16. The columns of an \( n \times n \) invertible matrix from a basis for \( \mathbb{R}^n \).
17. If matrix \( A \) is row equivalent to matrix \( B \), then \( \text{Ker}(A) = \text{Ker}(B) \).
18. Two similar matrices have the same eigenvectors.