**Pre-Midterm Review Topics**

- Be able to put complex numbers in exponential form
- Know how to calculate roots and powers of complex numbers
- Know how to compute matrix-vector and matrix-matrix products
- Reduction of matrices to echelon form
- Identify matrices in echelon form
- Identify pivot and free variables from a matrix in echelon form
- Know how to determine existence and types of solutions
- Reduction to row canonical form
- Calculation of matrix inverse
- Know how to use matrix inverse to solve a system of equations
- Understand linear dependence and independence (also in relation to linear combinations)
- Know how to find a basis for a span of a given set of vectors
- Find a basis for rowsp(A) and colsp(A)
- Know how to determine the rank of a matrix
- Know how to determine the dim of a subspace spanned by a set of vectors
- Understand Rank-nullity theorem and how to deduce the dimension of the kernel of a matrix
- Know \( \dim(\mathbb{R}^n) = n \)
- Understand concepts of spans and subspaces
- Know how to determine if \( \text{span}(u_1, \ldots, u_m) = \mathbb{R}^n \)
- Know conditions for existence of a matrix inverse

**Post-Midterm Review Topics**

- Computing eigenvalues
- Computing eigenvectors
- Understand algebraic and geometric multiplicity of eigenvalues and relationship to diagonalizability of matrices
- Determining invertibility of a matrix
- Determining if a matrix is diagonalizable
- Diagonalization of a matrix
- Computing determinants
• \( \det(A) = \lambda_1 \lambda_2 \ldots \lambda_n \)

• Computing orthogonal complement of a vector subspace

• Computing dimension of an orthogonal complement of a subspace

• Computing distance between two vectors

• Computing inner dot product for two vectors

• Computing projection of vector onto a subspace given an orthogonal basis.

• Finding coefficients in expansion of a vector as a linear combination of an orthogonal basis

• Determine orthogonality of a set of vectors

• Least-squares approximation and uniqueness.

• The Gram-Schmidt Process.

• The Spectral Theorem for Symmetric Matrices

• Conceptual understanding of orthogonal decomposition of a vector.

**Additional Review Problems**

Determine whether the following statements are true or false

1. Two row equivalent matrices have the same rank

2. There exists a \(3 \times 2\) matrix with rank 3.

3. A homogeneous linear equation always has a solution

4. If \(v \in \mathbb{R}^n\), then \(-v\) is in the \(span\{v\}\)

5. Let \(u \in \mathbb{R}^n\) and \(v \in \mathbb{R}^n\); then \(span\{u, u - v\}\) contains \(v\).

6. If a \(6 \times 4\) matrix \(A\) has linearly independent columns, then the echelon form of \(A\) contains two zero rows.

7. There exist a \(3 \times 5\) matrix whose column space has dimension 4.

8. If a square matrix \(A\) has two identical columns, then \(\det(A) = 0\).

9. If \(\lambda = 0\) is an eigenvalue of the square matrix \(A\), then \(A\) is invertible.

10. Every invertible matrix is diagonalizable.

11. Every diagonalizable matrix is invertible.

12. The set of all solutions of a system of homogeneous equation with \(m\) equations and \(n\) unknowns is a subspace in \(\mathbb{R}^m\).

13. The set of all linear combinations of columns of an \(m \times n\) matrix is a subspace in \(\mathbb{R}^n\)
14. In general, the columns of an $n \times n$ matrix $A$ form a basis for $\text{colsp}(A)$.

15. The columns of an $n \times n$ invertible matrix form a basis for $\mathbb{R}^n$.

16. If matrix $A$ is row equivalent to matrix $B$, then $\text{Ker}(A) = \text{Ker}(B)$.

17. Two similar matrices have the same eigenvectors.