Pre-Midterm Review Topics

- Be able to put complex numbers in exponential form
- Know how to calculate roots and powers of complex numbers
- Know how to compute matrix-vector and matrix-matrix products
- Reduction of matrices to echelon form
- Identify matrices in echelon form
- Identify pivot and free variables from a matrix in echelon form
- Know how to determine existence and types of solutions
- Reduction to row canonical form
- Calculation of matrix inverse
- Know how to use matrix inverse to solve a system of equations
- Understand linear dependence and independence (also in relation to linear combinations)
- Know how to find a basis for a span of a given set of vectors
- Find a basis for rowsp(A) and colsp(A)
- Know how to determine the rank of a matrix
- Know how to determine the dim of a subspace spanned by a set of vectors
- Understand Rank-nullity theorem and how to deduce the dimension of the kernel of a matrix
- Know $\dim(\mathbb{R}^n) = n$
- Understand concepts of spans and subspaces
- Know how to determine if $span(u_1, \ldots, u_m) = \mathbb{R}^n$
- Know conditions for existence of a matrix inverse

Post-Midterm Review Topics

- Computing eigenvalues
- Computing eigenvectors
- Understand algebraic and geometric multiplicity of eigenvalues and relationship to diagonalizability of matrices
- Determining invertibility of a matrix
- Determining if a matrix is diagonalizable
- Diagonalization of a matrix
- Computing determinants

- $\det(A) = \lambda_1 \lambda_2 \ldots \lambda_n$
- Computing orthogonal complement of a vector subspace
- Computing dimension of an orthogonal complement of a subspace
- Computing distance between two vectors
- Computing inner dot product for two vectors
- Computing projection of vector onto a subspace given an orthogonal basis.
- Finding coefficients in expansion of a vector as a linear combination of an orthogonal basis
- Determine orthogonality of a set of vectors
- Least-squares approximation and uniqueness.
- The Gram-Schmidt Process.
- The Spectral Theorem for Symmetric Matrices
- Conceptual understanding of orthogonal decomposition of a vector.

Additional Review Problems

Determine whether the following statements are true or false

1. Two row equivalent matrices have the same rank
2. There exists a 3×2 matrix with rank 3.
3. A homogeneous linear equation always has a solution
4. If $v \in \mathbb{R}^n$, then $-v$ is in the $span\{v\}$
5. Let $u \in \mathbb{R}^n$ and $v \in \mathbb{R}^n$; then $span\{u, u - v\}$ contains v.
6. If a 6×4 matrix A has linearly independent columns, then the echelon form of A contains two zero rows.
7. There exist a 3×5 matrix whose column space has dimension 4.
8. If a square matrix A has two identical columns, then $\det(A) = 0$.
9. If $\lambda = 0$ is an eigenvalue of the square matrix A, then A is invertible.
10. Every invertible matrix is diagonalizable.
11. Every diagonalizable matrix is invertible.
12. The set of all solutions of a system of homogeneous equation with m equations and n unknowns is a subspace in $\mathbb{R}^m$.
13. The set of all linear combinations of columns of an $m \times n$ matrix is a subspace in $\mathbb{R}^n$

14. In general, the columns of an $n \times n$ matrix A form a basis for $colsp(A)$.

15. The columns of an $n \times n$ invertible matrix from a basis for $\mathbb{R}^n$.

16. If matrix A is row equivalent to matrix B, then $Ker(A) = Ker(B)$.

17. Two similar matrices have the same eigenvectors.