AMS10 HW1 Grading Rubric

1. (faithful effort credit 16 pts- 2pts each) A vector space is a set V on which two operations, vector addition and scalar multiplication are defined. Find and cite a credible source that defines these operations. What are the conditions that must be satisfied?

The vector addition operation (+) must satisfy the following conditions:

(1) Commutative law: For all vectors $\vec{u}$ and $\vec{v}$ in V, $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

(2) Associative law: For all vectors $\vec{u}$,$\vec{v}$, and $\vec{w}$ in V, $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$

(3) Additive identity: The set V contains an additive identity element, denoted by a **0**, such that for any vector $\vec{v}$ in V, $\vec{u} + \mathbf{0} = \mathbf{0} + \vec{u}$

(4) Additive inverses: For each vector $\vec{v}$ in V, the equations $\vec{v} + \vec{x} = 0$ and $\vec{x} + \vec{v} = 0$ have a solution $\vec{x}$ in V, called an additive inverse of $\vec{v}$, and denoted by $-\vec{v}$.

Note: *Closure*: If $\vec{u}$ and $\vec{v}$ are any vectors in V, then the sum $\vec{u} + \vec{v}$ belongs to V.

The scalar multiplication operation $(\cdot)$ is defined between real numbers (or scalars) and vectors, and must satisfy the following conditions:

(5) Distributive law: For all real numbers c and all vectors $\vec{u}, \vec{v}$ in V, was $c \cdot (\vec{u} + \vec{v}) = c \cdot \vec{u} + c \cdot \vec{v}$

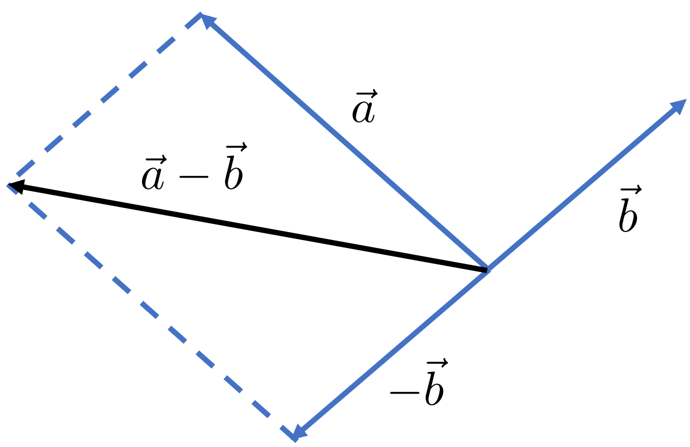
(6) Distributive law: For all real numbers c, d and all vectors $\vec{v}$ in V, is $(c + d) \cdot \vec{v} = c \cdot \vec{v} + d \cdot \vec{v}$

(7) Associative law: For all real numbers c, d and all vectors $\vec{v}$ in V, $c \cdot (d \cdot \vec{v}) = (cd) \cdot \vec{v}$

(8) Unitary law: For all vectors $\vec{v}$ in V, $1 \cdot \vec{v} = \vec{v}$

Note: *Closure*: If $\vec{v}$ is any vector in V, and c is any real number, then the product $c \cdot \vec{v}$ belongs to V.

2. (faithful effort credit 14 pts) Draw the vector $\vec{a} - \vec{b}$ using the parallelogram law. Note that $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$,


$\vec{a}$
$\vec{a} - \vec{b}$
$\vec{b}$
$-\vec{b}$

3. a. (14pts - 7pts each) Calculate the absolute value $|z|$ for the following complex numbers

i. $z = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

ii. $z = 5 \cdot \frac{-\sqrt{2}}{2} + i5 \cdot \frac{\sqrt{2}}{2}$

Solution:

i. $|z| = 1$

ii. $|z| = 5$

b. (14pts - 7pts each) Write down the complex exponential form of

i. $z = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

ii. $z = 5 \cdot \frac{-\sqrt{2}}{2} + i5 \cdot \frac{\sqrt{2}}{2}$

(Hint: there are an infinite number of representations)
Solution:

i. $z = e^{i(\frac{\pi}{3}+2\pi k)}$ for $k = 0, \pm 1, \pm 2, \ldots$

ii. $z = 5e^{i(\frac{3\pi}{4}+2\pi k)}$ for $k = 0, \pm 1, \pm 2, \ldots$

c. (27 pts)Use the complex exponential form of $z = 10\frac{\sqrt{3}}{2} + i10\frac{1}{2}$ to show that $x^2 = 10\frac{\sqrt{3}}{2} + i10\frac{1}{2}$ has only two distinct solutions.

We have

$$z^2 = 10\frac{\sqrt{3}}{2} + i10\frac{1}{2} \tag{1}$$

$$z^2 = 10e^{i(\frac{\pi}{6}+2\pi k)} \text{(10pts if you got this far)} \tag{2}$$

$$(z^2)^{1/2} = (10e^{i(\frac{\pi}{6}+2\pi k)})^{1/2} \tag{3}$$

$$z = \sqrt{10}e^{i(\frac{\pi}{12}+\pi k)} \tag{4}$$

for $k = 0, \pm 1, \pm 2, \ldots$ (20 pts for this far)

Note that

$$\sqrt{10}e^{i(\frac{\pi}{12}+\pi)} = \sqrt{10}e^{i(\frac{\pi}{12}+3\pi)} = \sqrt{10}e^{i(\frac{\pi}{12}+5\pi)} = \sqrt{10}e^{i(\frac{\pi}{12}+(2k+1)\pi)}$$

and

$$\sqrt{10}e^{i(\frac{\pi}{12}+2\pi)} = \sqrt{10}e^{i(\frac{\pi}{12}+4\pi)} = \sqrt{10}e^{i(\frac{\pi}{12}+6\pi)} = \sqrt{10}e^{i(\frac{\pi}{12}+2k\pi)}$$

and so there are only two unique solutions (full credit).

4. (faithful effort credit 15pts- 5pts each) Construct the following vectors in Matlab:

a. A 2x1 vector with all 1's in the entries and define it u.

b. A 2x1 vector with the first element 4 and the second element 3 and define it v

c. What is 5u+v? Verify using Matlab

Write down using Matlab syntax the exact expression you would type into the command window for a-c.

Solution:

a. u=[1;1];

b. v=[4;3];

c. 5*u+v , answer: 5u+v=[9;8]