AMS10 HW2

Problem 1.Simplifying fractions of complex numbers. Suppose we have a complex number expressed as the division of two distinct complex numbers of the form

$$z = \frac{a_1 + ib_1}{a_2 + ib_2}$$

Then a method of finding the real and imaginary component involves the complex conjugate of the denominator in order to remove any imaginary numbers from the denominator. Recall that $z\bar{z} = |z|^2$ is a real non-negative number. The following is an illustration of this

$$z = \frac{a_1 + ib_1}{a_2 + ib_2} \tag{5}$$
$$= \frac{1}{1} \cdot \frac{a_1 + ib_1}{a_2 + ib_2} \tag{6}$$
$$= \frac{a_2 - ib_2}{a_2 - ib_2} \cdot \frac{a_1 + ib_1}{a_2 + ib_2} \tag{7}$$
$$= \frac{(a_2 - ib_2)(a_1 + ib_1)}{a_2^2 + b_2^2}. \tag{8}$$

Apply this method to find the real and imaginary components of the following expressions (don't just plug in values but show work):

$$\text{a. } \frac{3+i}{2+i} \tag{9}$$
$$\text{b. } \frac{1+i}{2-i} \tag{10}$$
$$\text{c. } \frac{1-2i}{2+2i} \tag{11}$$
$$\text{d. } \frac{-1-2i}{1+2i} \tag{12}$$
$$\text{e. } \frac{-1+2i}{5+3i} \tag{13}$$
$$\tag{14}$$

Problem 2. Division of complex numbers using the exponential form. Suppose you are given two complex numbers in the form $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$, then the division is given by

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\frac{e^{i\theta_1}}{e^{i\theta_2}} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$$

which is equivalent to

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right).$$

a. Find the argument (or phase) for the numerator and denominator of (a)-(e) in Problem 1. Find the angle using arctan and applying the appropriate adjustment. Show work. Verify the answer using the Matlab command **angle** (e.g. angle(1+i)).

b. Find the modulus (or absolute value) for the numerator and denominator of (a)-(e) in Problem 1. You may work out by hand or use the Matlab command **abs** (e.g. abs(1+i)). You can also work out by hand and verify using Matlab.

c. Calculate the real and imaginary component of (a)-(e) using the complex exponential form and verify that it is equal to the results found in Problem 1. Recall, two complex numbers $a + ib$ and $c + id$ are equal if and only if $a = c$ and $b = d$.

Problem 3. Given the complex number $z = -1 + 2i$ find the m distinct m-th roots ($z^{1/m}$) for

a. m=10

b. m=45

c. m=100

It is ok to leave in exponential form.

Problem 4.Calculate the following powers of $z = -1 + 2i$

a. z^{10}

b. z^{15}

c. z^{200}

It is ok to leave in exponential form.

Problem 5. Consider the following linear system of equations

$$x_1 + 5x_2 + 2x_3 = 3 \quad (15)$$

$$x_2 + 5x_3 = 5 \quad (16)$$

$$2x_1 + x_2 + x_3 = 1. \quad (17)$$

a. Apply the following sequence of elementary operations

1. $L_2 \leftrightarrow L_3$
2. $-2L_1 + L_2 \rightarrow L_2$
3. $\frac{1}{9}L_2 + L_3 \rightarrow L_3$.

You should end up in echelon form (in fact you will have triangular form)

b. Solve for x_1, x_2, and x_3.

c. Does this system have no solution, one solution, or infinite solutions. How can you be sure? (hint: apply theorem for systems in echelon form)