AMS10 HW3

*On all problems you can use Matlab to verify your answer but you must show work unless otherwise indicated.

Problem 1 Consider the following set of vectors

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \right\} = \{v_1, v_2\}.$$

$Span(\{v_1, v_2\})$ is the set of all linear combinations of $\{v_1, v_2\}$:

$$k_1 v_1 + k_2 v_2 : k_1, k_2 \in \mathbb{R}.$$

Let $\vec{b} = [a, b, c]^T \in Span(S)$. Note, we can rewrite this as

$$[v_1, v_2] \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

a. Put the augmented matrix $[v_1, v_2|\vec{b}]$ in row canonical form, carrying through variables a, b, c.
b. Define the set of vectors that is not in the span of $\{v_1, v_2\}$ Hint: check for consistency.

Problem 2 Consider the following system

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 1 \\ x_1 + x_3 &= 0 \end{aligned}$$

a. Find the row canonical form of the corresponding augmented matrix. How many solutions, if any, does this system have?
b. Write down the reduced system (obtained after reduction to row canonical form) as a linear combination of vectors:

$$\vec{a}_1 x_1 + \vec{a}_2 x_2 + \vec{a}_3 x_3 = \vec{b}.$$

What is $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{b}$?
c. Does there exist scalars k_1, k_2, k_3 such that

$$k_1 \vec{a}_1 + k_2 \vec{a}_2 + k_3 \vec{a}_3 = 0?$$

If so, give an example. Is the solution unique?
d. Are the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ linearly dependent or independent? Why?
e. Does there exist a different vector $\vec{b}$ for which no solution exists? Why?

Problem 3 Consider the following vectors

$$v_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ -2 \\ -4 \end{bmatrix}, v_3 = \begin{bmatrix} -2 \\ 6 \\ 14 \end{bmatrix}.$$

What is the rank of matrix $A = [v_1, v_2, v_3]$? What does this say about the dimension of $colsp(A) = span(v_1, v_2, v_3)$?

Problem 4 Let matrix A be composed of

$$v_1 = \begin{bmatrix} 9 \\ -4 \\ 5 \\ 5 \end{bmatrix}, v_2 = \begin{bmatrix} -19 \\ 2 \\ -19 \\ -2 \end{bmatrix}, v_3 = \begin{bmatrix} -34 \\ 28 \\ -2 \\ -36 \end{bmatrix}, v_4 = \begin{bmatrix} 1 \\ 6 \\ 9 \\ -8 \end{bmatrix}.$$

In this problem you may use Matlab to reduce matrices to row canonical form.
a. Reduce the matrix $A = [v_1, v_2, v_3, v_4]$ to row canonical form. Are the vectors v_1, v_2, v_3, v_4 linearly dependent or independent?
b. Reduce the matrix $A = [v_1, v_2]$ to row canonical form. Are the vectors v_1, v_2 linearly dependent or independent? How does the row canonical form compare to the first two columns for the reduced matrix in part a.
c. Reduce the matrix $A = [v_1, v_2, v_3]$ to row canonical form. Are the vectors v_1, v_2, v_3 linearly dependent or independent? How does the row canonical form compare to the first three columns for the reduced matrix in part a.
d. Reduce the matrix $A = [v_3, v_4]$ to row canonical form. Are the vectors v_3, v_4 linearly dependent or independent?
e. Give a basis for the span$\{v_1, v_2, v_3, v_4\}$.
f. What is the $dim(colsp(A))$? Will any subset of three vectors always be linearly dependent?

Problem 5 Consider the following set of vectors

$$v_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ -3 \\ -2 \end{bmatrix}, v_4 = \begin{bmatrix} -2 \\ -2 \\ 2 \\ -3 \\ 8 \end{bmatrix}.$$

a.Find the dimension of the subspace spanned by the vectors
b.Find a basis for the subspace spanned by the vectors
c.What is the rank of the matrix $A = [v_1, v_2, v_3, v_4]$?