AMS10 HW4

*On all problems you can use Matlab to verify your answer but you must show work.

Problem 1 Let

$$A = \begin{bmatrix} 1 & -4 \\ 2 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}, \ C = \begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix}$$

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \ c = 2$$

a. Show that $A\vec{v}_1 + A\vec{v}_2 = A(\vec{v}_1 + \vec{v}_2)$
b. Show that $AB \neq BA$
c. Show that $A\vec{v}_1 + B\vec{v}_1 = (A + B)\vec{v}_1$
d. Show that $A(c\vec{v}_1) = c(A\vec{v}_1) = (cA)\vec{v}_1$. Note that $[cA]_{ij} = c[A]_{ij}$.
e. Show that $A(B + C) = AB + AC$.
f. Show that $(A + B)C = AC + BC$.
g. Show that $c(AB) = (cA)B = (AB)c = A(Bc)$.

Problem 2 For the following pairs of A and $\vec{v}$, first determine whether the product $A\vec{v}$ is defined. If it is, what is the expected dimension of the product? Calculate the product and verify the dimension.

a.

$$A = [1\ 2\ 3\ 4], \ \vec{v} = \begin{bmatrix} 1 \\ 8 \\ 0 \\ 2 \end{bmatrix}$$

b.

$$A = \begin{bmatrix} 1 & -2 & 3 & 4 \\ 0 & 1 & 8 & -3 \end{bmatrix}, \ \vec{v} = \begin{bmatrix} 1 \\ 8 \\ 0 \\ 2 \end{bmatrix}$$

c.

$$A = \begin{bmatrix} 1 & 4 \\ 0 & -3 \\ 2 & 1 \end{bmatrix}, \ \vec{v} = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$$

Problem 3 For the following pairs of A and B, first determine whether the product AB is defined. If it is, what is the expected dimension of the product? Calculate the product and verify the dimension.

a.

$$A = \begin{bmatrix} 1 & -2 & 0 & 1 \\ 3 & 4 & -1 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 0 & 7 \\ 2 & -4 & 9 \\ -1 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

b.

$$A = \begin{bmatrix} 1 & -4 & 3 \\ 0 & 2 & 1 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -4 & 1 & 5 \\ -2 & -3 & 1 \end{bmatrix}$$

c.

$$A = \begin{bmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 5 & 2 \\ 3 & 7 & 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 0 \\ 0 & -1 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

Problem 4 Consider the following system

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 1 \\ x_1 + x_3 &= 0 \end{aligned}$$

a. Find the row canonical form of the corresponding augmented matrix. How many solutions, if any, does this system have?
b. Write down the reduced system (obtained after reduction to row canonical form) as a linear combination of vectors:

$$\vec{a}_1 x_1 + \vec{a}_2 x_2 + \vec{a}_3 x_3 = \vec{b}.$$

What is $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{b}$?
c. Does there exist scalars k_1, k_2, k_3 such that

$$k_1 \vec{a}_1 + k_2 \vec{a}_2 + k_3 \vec{a}_3 = 0?$$

If so, give an example. Is the solution unique?
d. Are the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ linearly dependent or independent? Why?
e. Does there exist a different vector $\vec{b}$ for which no solution exists? Why?
f. Put in matrix multiplication form $A\vec{x} = \vec{b}$. Augment the system with

$$0 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 = 0,$$

which doesn't change the system and put it in matrix multiplication form again. What happens if you type in inv(A) into Matlab in both cases? Why?

Problem 5 Consider the following system

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 1 \\ x_1 + x_3 &= 0 \\ 2x_1 + x_2 + 3x_3 &= 1 \end{aligned}$$

a. Is the system above linearly dependent or independent? Show using any method.
b. Put the system in matrix multiplication form $A\vec{x} = \vec{b}$. Does the inverse A^{-1} exist? Why or why not?
c. Use Matlab to find the row canonical form and write the output. How does the row canonical form compare to that of the system above? Explain why it's the same or different.

Problem 6 Consider the following system

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 1 \\ x_1 + x_3 &= 0 \\ 2x_1 + x_2 + 3x_3 &= 1 \\ x_1 - 5x_2 &= 0 \end{aligned}$$

a. There are more equations than unknowns. More equations than unknowns always implies a linearly dependent set if not inconsistent. Apply Gaussian elimination steps (towards row echelon form) until the system reduces to 3 equations. Note that the remaining equations in echelon form are always linearly independent. No need to fully reduce system.
b. Put the partially reduced system (3 equations) in matrix multiplication form $A\vec{x} = \vec{b}$. What is A, $\vec{x}$, and $\vec{b}$?
c. Calculate the inverse A^{-1} by reducing the left side of the augmented matrix $[A|I]$ to row canonical form, where I is the identity matrix. Solution can be verified using the function inv in Matlab.
d. Find the solution $\vec{x}$ using the calculated inverse. Show work.
e. Verify your answer by applying the Matlab function rref on the augmented matrix of the original system to show that the answers are the same. Write down the output.