AMS10 HW5

*On all problems you can use Matlab to verify your answer but you must show work unless otherwise indicated.

Problem 1 Consider the following set of vectors

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \right\} = \{v_1, v_2\}.$$

$Span(\{v_1, v_2\})$ is the set of all linear combinations of $\{v_1, v_2\}$:

$$k_1 v_1 + k_2 v_2 : k_1, k_2 \in \mathbb{R}.$$

Let $\vec{b} = [a, b, c]^T \in Span(S)$. Note, we can rewrite this as

$$[v_1, v_2] \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

a. Put the augmented matrix $[v_1, v_2|\vec{b}]$ in row canonical form, carrying through variables a, b, c.
b. What constraints if any are placed on the variables a, b, c.
c. Define the set of vectors that is not in the span of $\{v_1, v_2\}$
d. Define the set of vectors in the span of $\{v_1, v_2\}$

Problem 2 Consider the following vectors

$$v_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ -2 \\ -4 \end{bmatrix}, v_3 = \begin{bmatrix} -2 \\ 6 \\ 14 \end{bmatrix}.$$

a. What is the rank of matrix $A = [v_1, v_2, v_3]$? What does this say about the dimension of $colsp(A)$ (also the image of A and span(v_1, v_2, v_3)?
b. Note that solving $A\vec{x} = \vec{0}$, gives the linear combination of the vectors v_1, v_2, v_3 that maps to the zero vector. Hence, it gives vectors $\vec{x}$ in the nullspace of A. Use the reduced matrix of A (you may use rref([A|0]) in Matlab) to find a vector $\vec{x}$ that satisfies the system of equations and verify it is in the null space of A by showing $A\vec{x} = 0$.
c. Use the Rank-Nulliity theorem to determine the dimension of Ker(A). Does this agree with part b?

Problem 3 Let matrix A be composed of

$$v_1 = \begin{bmatrix} 9 \\ -4 \\ 5 \\ 5 \end{bmatrix}, v_2 = \begin{bmatrix} -19 \\ 2 \\ -19 \\ -2 \end{bmatrix}, v_3 = \begin{bmatrix} -34 \\ 28 \\ -2 \\ -36 \end{bmatrix}, v_4 = \begin{bmatrix} 1 \\ 6 \\ 9 \\ -8 \end{bmatrix}.$$

In this problem you may use Matlab to reduce matrices to row canonical form.

a. Reduce the matrix $A = [v_1, v_2, v_3, v_4]$ to row canonical form. Are the vectors v_1, v_2, v_3, v_4 linearly dependent or independent?
b. Reduce the matrix $A = [v_1, v_2]$ to row canonical form. Are the vectors v_1, v_2 linearly dependent or independent? How does the row canonical form compare to the first two columns for the reduced matrix in part a.
c. Reduce the matrix $A = [v_1, v_2, v_3]$ to row canonical form. Are the vectors v_1, v_2, v_3 linearly dependent or independent? How does the row canonical form compare to the first three columns for the reduced matrix in part a.
d. Reduce the matrix $A = [v_3, v_4]$ to row canonical form. Are the vectors v_3, v_4 linearly dependent or independent?
e. Will any subset of three vectors always be linearly dependent?
f. Give a basis for the span$\{v_1, v_2, v_3, v_4\}$.

Problem 4 Consider the following set of vectors

$$v_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ -3 \\ -2 \end{bmatrix}, v_4 = \begin{bmatrix} -2 \\ -2 \\ 2 \\ -3 \\ 8 \end{bmatrix}.$$

a.Find the dimension of the subspace spanned by the vectors
b.Find a basis for the subspace spanned by the vectors
c.What is the rank of the matrix $A = [v_1, v_2, v_3, v_4]$?

Midterm Review Topics

- Be able to put complex numbers in exponential form
- Know how to calculate roots and powers of complex numbers
- Know how to compute matrix-vector and matrix-matrix products
- Reduction of matrices to echelon form
- Identify matrices in echelon form
- Identify pivot and free variables from a matrix in echelon form
- Know how to determine existence and types of solutions
- Reduction to row canonical form
- Calculation of matrix inverse
- Know how to use matrix inverse to solve a system of equations
- Understand linear dependence and independence (also in relation to linear combinations)
- Know how to find a basis for a span of a given set of vectors

- Find a basis for rowsp(A) and colsp(A)
- Know how to determine the rank of a matrix
- Know how to determine the dim of a subspace spanned by a set of vectors
- Understand Rank-nullity theorem and how to deduce the dimension of the kernel of a matrix
- Know $\dim(\mathbb{R}^n) = n$
- Understand concepts of spans and subspaces
- Know how to determine if $span(u_1, \ldots, u_m) = \mathbb{R}^n$
- Know conditions for existence of a matrix inverse

Additional Review Problems

Example 1 How to check if $\vec{b}$ is in the span of $a_1, a_2, \ldots, a_n$. Hint: If it's in the span there there is a linear combination such that

$$b = k_1 a_1 + \cdots + k_n a_n$$

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Example 2 Is it possible that matrix $A \in \mathbb{R}^{11\times 9}$ has a pivot position in every row?

Example 3 Suppose $a_1, \ldots, a_n \in \mathbb{R}^m$. How to check if span$(a_1, \ldots, a_n)$=$\mathbb{R}^n$.

Example 4 How to check if $A\vec{x} = 0$ has a non-trivial solution (i.e. not $\vec{x} = 0$)

Example 5. Suppose $A \in \mathbb{R}^{11\times 15}$, does $A\vec{x} = 0$ have a non-trivial solution?

Example 6 How to check if a set of vectors is linearly independent.

Example 7. Under what conditions Is $\{v_1, v_2, 0\}$ linearly dependent?

Example 8 Is $\{2\vec{u}, 7\vec{u}\}$ linearly dependent?

Example 9 Suppose $a_1, \ldots, a_n \in \mathbb{R}^m$ and $n > m$. Is the set linearly dependent?

Example 10 Suppose $A \in \mathbb{R}^{3\times 5}$ and $B \in \mathbb{R}^{4\times 3}$. Is AB well defined? Is BA well defined? Is A^7 well defined? Is $(AB)^7$ well defined?