AMS10 HW5

Problem 1

a. Eigenvalues of the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 3 & 1 \end{bmatrix}$$

can be calculated by solving the system $\det(sI - A) = 0$. Solve for s:

$$\det\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 5 & 4 \\ 3 & 1 \end{bmatrix}\right) = 0.$$

You may use the quadratic formula. The general quadratic equation $ax^2 + bx + c = 0$ has the following solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

b. Find the determinant of the matrix and verify that $\det(A) = \lambda_1\lambda_2$.
c. Find a basis for the Image of A.
d. Find a basis for the Kernel of A.
e. Does A^{-1} exist?
f. For each of the eigenvalues calculate the matrix $M_i = A - \lambda_i I$.
g. Find the basis for the Kernel of each of the matrices M_1 and M_2 found in part f. These are the eigenvectors.

Problem 2 Consider the following matrix

$$A = \begin{bmatrix} -1 & 2 & 4 \\ 0 & 5 & 2 \\ 2 & -1 & 9 \end{bmatrix}$$

For this problem you may use Matlab. **a.** Calculate the eigenvalues of A.
b. Calculate the eigenvalues of the row canonical form of A. How do the eigenvalues compare to those of A?
c. Calculate the eigenvalues of A^T. How do the eigenvalues compare to those of A?

Problem 3 Consider the following matrix

$$A = \begin{bmatrix} -1 & 2 \\ 0 & 5 \end{bmatrix}$$

a. What method can be used to show that this matrix is diagonalizable?
b. Calculate by hand the eigenvalues and associated eigenvectors.
c. Find the matrix P such that $A = PDP^{-1}$.
d. Derive the relation $D = P^{-1}AP$ from $A = PDP^{-1}$. (Hint: $PP^{-1} = P^{-1}P = I$)
e. Use the matrix P to compute by hand the diagonal matrix $D = P^{-1}AP$ and verify that the elements along the diagonal are indeed the eigenvalues of the matrix A.
f. Using the relation $A = PDP^{-1}$, compute by hand A^3 by computing $(PDP^{-1})^3$. (Hint: $PP^{-1} = P^{-1}P = I$)
g. Using the results from **f** determine the eigenvalues of A^3 without deriving the characteristic polynomial.

Problem 4 Consider the following matrix

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 3 & 1 \\ 3 & 0 & 1 \end{bmatrix}$$

a. Find the characteristic polynomial $p(s)$ of the matrix A.
b. Show that the matrix A satisfies the characteristic polynomial. That is $p(A)$ gives the 3×3 zero matrix. You may use Matlab to compute after setting up the problem.
c. Compute the eigenvalues of A. You may use Matlab here.
d. For each of the eigenvalues $\lambda_1, \lambda_2, \lambda_3$, compute the associated eigenvectors. You may use Matlab. Show that v_i is in the null space of $A - \lambda_i I$ by computing $(A - \lambda_i I)v_i$, where v_i is the eigenvector associated with the eigenvalue λ_i. Show that v_i is invariant under the transformation A by computing Av_i.

Problem 5 Consider the following matrix

$$A = \begin{bmatrix} -5 & 2 & -1 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

a. Compute the eigenvalues by hand. (Hint: Use the transpose A^T to compute the eigenvalues using the method discussed in class for upper triangular matrices. This eases computation). How do the eigenvalues compare to the diagonal entries of the matrix? Note, this property always holds true for an upper triangular matrix such as A (and a lower triangular matrix such as A^T).
b. For any repeated eigenvalues, find the eigenvectors using rref in Matlab or by hand if you wish. Are the eigenvectors linearly independent? The number of times the eigenvalue is repeated is its algebraic multiplicity. What is the algebraic multiplicity for repeated eigenvalues? The geometric multiplicity is given by the maximum number of linearly independent associated eigenvectors. What is the geometric multiplicity for repeated eigenvalues?
c. Consider the matrix

$$B = \begin{bmatrix} -5 & 2 & -1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}.$$

What is the algebraic multiplicity and geometric multiplicity for any repeated eigenvalues? You may use Matlab. Is the matrix B diagonalizable?