# AMS10 HW6- Grading Rubric

**Automatic 10pts for submission**

Points on all problems given for effort.

**Problem 1 (30pts–5pts/each)** Given the matrices

$$A = \begin{bmatrix} 5 & 4 \\ -3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & 4 \\ 2 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & -3 & 1 \\ 1 & 1 & -2 \\ 0 & 5 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & -3 & 1 & 0 \\ 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 5 \\ 3 & -1 & 0 & 2 \end{bmatrix}$$

calculate the following determinants:

**a**. $\det(A) = 17$

**b**. $\det(B) = 19$

**c**. $\det(C) = 8$

**d**. $\det(BC) = \det(B)\det(C) = 19 \times 8 = 152$ (Hint: Can use results from b. and c.)

**e**. $\det(CB) = \det(C)\det(B) = \det(B)\det(C) = 19 \times 8 = 152$ (Hint: Can use results from b. and c.)

**f**. $\det(D) = -52$

You can verify answers using Matlab.

**Problem 2 (25pts–5pts/each)**

**a**. Find the determinant of the following matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 2 & 0 & 1 \end{bmatrix}$$

$\det(A) = 0$

**b**. Use Matlab to calculate the eigenvalues by using the matlab command **eig** and verify that at least one of the eigenvalues is zero.

The eigenvalues of A are $\lambda_1 = 4.6458, \lambda_2 = -.6458, \text{and } \lambda_3 = 0$.

**c.** Find a basis for the Image of A. Recall, the image of A is the span of the column vectors. Use one of the methods in Lecture 10. The number of elements in the basis should equal the number of nonzero eigenvalues.

$$rref(A) = \begin{bmatrix} 1 & 0 & .5 \\ 0 & 1 & .25 \\ 0 & 0 & 0 \end{bmatrix}$$

The pivot points are in the first two columns so we take the first two columns of the matrix A as the basis

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \right\}$$

**d.** Find a basis for the Kernel of A. The number of elements in the basis should equal the number of zero eigenvalues.

Using the row canonical form we solve $Ax = 0$

$$x_1 + .5x_3 = 0 \to x_1 = -.5x_3$$
$$x_2 + .25x_3 = 0 \to x_2 = -.25x_3$$

A solution vector has the form

$$\begin{bmatrix} -.5x_3 \\ -.25x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -.5 \\ -.25 \\ 1 \end{bmatrix}$$

and so a basis is

$$\left\{ \begin{bmatrix} -.5 \\ -.25 \\ 1 \end{bmatrix} \right\}$$

**e.** Does $A^{-1}$ exist?

No, because the matrix $A$ is singular.

**Problem 3. (35pts–5pts/each)**

**a**. Eigenvalues of the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 3 & 1 \end{bmatrix}$$

in Problem 1 can be calculated by solving the system $\det(sI - A) = 0$. Solve for $s$:

$$\det\left( \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 5 & 4 \\ 3 & 1 \end{bmatrix} \right) = 0.$$

You may use the quadratic formula. The general quadratic equation $ax^2 + bx + c = 0$ has the following solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$\det\left( \begin{bmatrix} s-5 & -4 \\ -3 & s-1 \end{bmatrix} \right) = (s-5)(s-1) - 12 = s^2 - 6s - 7.$$

$$\lambda_{1,2} = \frac{6 \pm \sqrt{6^2 - 4(-7)}}{2} \to \lambda_1 = 7, \lambda_2 = -1$$

**b**. Find the determinant of the matrix and verify that $\det(A) = \lambda_1\lambda_2$

$$\det(A) = 5(1) - 4(3) = 5 - 12 = -7$$

**c.** Find a basis for the Image of A.

$$rref(A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \end{bmatrix} \right\}$$

**d.** Find a basis for the Kernel of A.

The matrix is full rank and there is no zero eigenvalue so the Kernel of A is $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

**e.** Does $A^{-1}$ exist?

Yes, the matrix is full rank and invertible and so $A^{-1}$ exists.

**f.** For each of the eigenvalues calculate the matrix $M_i = A - \lambda_i I$

$$M_1 = A - 7I = \begin{bmatrix} -2 & 4 \\ 3 & -6 \end{bmatrix}$$

$$M_2 = A + I = \begin{bmatrix} 6 & 4 \\ 3 & 2 \end{bmatrix}$$

**g.** Find the basis for the Kernel of each of the matrices $M_1$ and $M_2$ found in part f. These are the eigenvectors.

$$rref(M_1) = \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \rightarrow \left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$$

$$rref(M_2) = \begin{bmatrix} 1 & 2/3 \\ 0 & 0 \end{bmatrix} \rightarrow \left\{ \begin{bmatrix} -2/3 \\ 1 \end{bmatrix} \right\}$$