AMS10 HW6

Problem 1 Given the matrices

$$A = \begin{bmatrix} 5 & 4 \\ -3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & 4 \\ 2 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & -3 & 1 \\ 1 & 1 & -2 \\ 0 & 5 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & -3 & 1 & 0 \\ 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 5 \\ 3 & -1 & 0 & 2 \end{bmatrix}$$

calculate the following determinants:

a. $\det(A)$
b. $\det(B)$
c. $\det(C)$
d. $\det(BC)$ (Hint: Can use results from b. and c.)
e. $\det(CB)$ (Hint: Can use results from b. and c.)
f. $\det(D)$

You can verify answers using Matlab.

Problem 2

a. Find the determinant of the following matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 2 & 0 & 1 \end{bmatrix}$$

b. Use Matlab to calculate the eigenvalues by using the matlab command **eig** and verify that at least one of the eigenvalues is zero.
c. Find a basis for the Image of A. Recall, the image of A is the span of the column vectors. Use one of the methods in Lecture 10. The number of elements in the basis should equal the number of nonzero eigenvalues.
d. Find a basis for the Kernel of A. The number of elements in the basis should equal the number of zero eigenvalues.
e. Does A^{-1} exist?

Problem 3

a. Eigenvalues of the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 3 & 1 \end{bmatrix}$$

in Problem 1 can be calculated by solving the system $\det(sI - A) = 0$. Solve for s:

$$\det\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 5 & 4 \\ 3 & 1 \end{bmatrix}\right) = 0.$$

You may use the quadratic formula. The general quadratic equation $ax^2 + bx + c = 0$ has the following solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

b. Find the determinant of the matrix and verify that $\det(A) = \lambda_1\lambda_2$
c. Find a basis for the Image of A.
d. Find a basis for the Kernel of A.
e. Does A^{-1} exist?
f. For each of the eigenvalues calculate the matrix $M_i = A - \lambda_i I$
g. Find the basis for the Kernel of each of the matrices M_1 and M_2 found in part f. These are the eigenvectors.