AMS10 HW7

Problem 1 Consider the following matrix

$$A = \begin{bmatrix} -1 & 2 & 4 \\ 0 & 5 & 2 \\ 2 & -1 & 9 \end{bmatrix}$$

a. Calculate the eigenvalues of A.
b. Calculate the eigenvalues of the row canonical form of A. How do the eigenvalues compare to those of A?
c. Calculate the eigenvalues of A^T. How do the eigenvalues compare to those of A?
$*$ For this problem you may use Matlab

Problem 2 Consider the following matrix

$$A = \begin{bmatrix} -1 & 2 \\ 0 & 5 \end{bmatrix}$$

a. What method can be used to show that this matrix is diagonalizable?
b. Calculate by hand the eigenvalues and associated eigenvectors.
c. Find the matrix P such that $A = PDP^{-1}$.
d. Derive the relation $D = P^{-1}AP$ from $A = PDP^{-1}$. (Hint: $PP^{-1} = P^{-1}P = I$)
e. Use the matrix P to compute by hand the diagonal matrix $D = P^{-1}AP$ and verify that the elements along the diagonal are indeed the eigenvalues of the matrix A.
f. Using the relation $A = PDP^{-1}$, compute by hand A^3 by computing $(PDP^{-1})^3$. (Hint: $PP^{-1} = P^{-1}P = I$)
g. Using the results from **f** determine the eigenvalues of A^3 without deriving the characteristic polynomial.

Problem 3 Consider the following matrix

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 3 & 1 \\ 3 & 0 & 1 \end{bmatrix}$$

a. Find the characteristic polynomial $p(s)$ of the matrix A.
b. Show that the matrix A satisfies the characteristic polynomial. That is $p(A)$ gives the 3×3 zero matrix. You may use Matlab to compute after setting up the problem.
c. Compute the eigenvalues of A. You may use Matlab here.
d. For each of the three eigenvalues λ_i compute the eigenvalues of the matrix $M_i = A - \lambda_i I$ for $i = 1, 2, 3$. How do the eigenvalues compare to those in part c. You may use Matlab again after setting up the problem.
e. For each of the eigenvalues $\lambda_1, \lambda_2, \lambda_3$, compute the associated eigenvectors. You may use Matlab. Show that v_i is in the null space of $A - \lambda_i I$ by computing $(A - \lambda_i I)v_i$, where v_i is the eigenvector associated with the eigenvalue λ_i. Show that v_i is invariant under the transformation A by computing Av_i.

Problem 4 In two dimensions, the standard rotation matrix has the following form

$$A = \begin{bmatrix} sin(\theta) & \cos(\theta) \\ -\cos(\theta) & sin(\theta) \end{bmatrix},$$

which rotates column vectors by an angle θ.
a. For $\theta = \pi/4$ compute by hand the eigenvalues and eigenvectors. Plot the vector $x = [1, 0]^T$ and the resulting vector Ax after transformation.
b. For $\theta = \pi/2$ compute by hand the eigenvalues and eigenvectors. Plot the vector $x = [1, 0]^T$ and the resulting vector Ax after transformation.
c. Is any real vector invariant under this transformation?

Problem 5 Consider the following matrix

$$A = \begin{bmatrix} -5 & 2 & -1 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

a. Compute the eigenvalues by hand. (Hint: Use the transpose A^T to compute the eigenvalues using the method discussed in class. This eases computation). How do the eigenvalues compare to the diagonal entries of the matrix? Note, this property always holds true for an upper triangular matrix such as A (and a lower triangular matrix such as A^T).
b. For any repeated eigenvalues, find the eigenvectors by hand. Are the eigenvectors linearly independent? The number of times the eigenvalue is repeated is its algebraic multiplicity. What is the algebraic multiplicity for repeated eigenvalues? The geometric multiplicity is given by the maximum number of linearly independent associated eigenvectors. What is the geometric multiplicity for repeated eigenvalues?
c. Consider the matrix

$$B = \begin{bmatrix} -5 & 2 & -1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}.$$

What is the algebraic multiplicity and geometric multiplicity for any repeated eigenvalues? You may use Matlab. Is the matrix B diagonalizable?