AMS10 HW7

Problem 1 Let

$$A = \begin{bmatrix} 3/\sqrt{18} & 2/3 & 1/\sqrt{18} \\ -3/\sqrt{18} & 2/3 & 1/\sqrt{18} \\ 0 & -1/3 & 4/\sqrt{18} \end{bmatrix}. \tag{1}$$

a. Compute $A^T A$.
b. What is A^{-1}?

Problem 2 Consider the system $A\vec{x} = \vec{b}$ for

$$A = \begin{bmatrix} 0 & 0 & 4 \\ 2 & 3 & -1 \\ 0 & 1 & 8 \\ 2 & 4 & 7 \\ 4 & 7 & 6 \end{bmatrix}, \qquad \vec{b} = \begin{bmatrix} -1 \\ -3 \\ 0 \\ -2 \\ -5 \end{bmatrix}. \tag{2}$$

a. Find the least-square solution.
b. Let $W = colsp(A)$. Find an orthogonal basis using the Gram-Schmidt process.
c. Find $proj_W \vec{b}$ and show that it is equal to $A\hat{x}$, where $\hat{x}$ is the least-square solution.

Problem 3 Let

$$A = \begin{bmatrix} 5 & -4 & -2 \\ -4 & 5 & 2 \\ -2 & 2 & 2 \end{bmatrix} \tag{3}$$

a. Find the eigenvectors of A.
b. Find an orthogonal set in the eigenspace for any repeated eigenvalues.
c. Find P and D that orthogonally diagonalize A. Note that since P is an orthogonal matrix $P^{-1} = P^T$.