Problem 1 Let
\[ A = \begin{bmatrix}
\frac{3}{\sqrt{18}} & 2/3 & 1/\sqrt{18} \\
-3/\sqrt{18} & 2/3 & 1/\sqrt{18} \\
0 & -1/3 & 4/\sqrt{18}
\end{bmatrix}. \] (1)

a. Compute \( A^T A \).
b. What is \( A^{-1} \)?

Problem 2 Consider the system \( A\vec{x} = \vec{b} \) for
\[ A = \begin{bmatrix}
0 & 0 & 4 \\
2 & 3 & -1 \\
0 & 1 & 8 \\
2 & 4 & 7 \\
4 & 7 & 6
\end{bmatrix}, \quad \vec{b} = \begin{bmatrix}
-1 \\
-3 \\
0 \\
-2 \\
-5
\end{bmatrix}. \] (2)

a. Find the least-square solution.
b. Let \( W = \text{colsp}(A) \). Find an orthogonal basis using the Gram-Schmidt process.
c. Find \( \text{proj}_W \vec{b} \) and show that it is equal to \( A\hat{x} \), where \( \hat{x} \) is the least-square solution.

Problem 3 Let
\[ A = \begin{bmatrix}
5 & -4 & -2 \\
-4 & 5 & 2 \\
-2 & 2 & 2
\end{bmatrix}. \] (3)

a. Find the eigenvectors of \( A \).
b. Find an orthogonal set in the eigenspace for any repeated eigenvalues.
c. Find \( P \) and \( D \) that orthogonally diagonalize \( A \). Note that since \( P \) is an orthogonal matrix \( P^{-1} = P^T \).