

AMS10 HW8

Problem 1 Let

$$u = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, v = \begin{bmatrix} -3 \\ 2 \\ 2 \end{bmatrix}.$$

Compute the following quantities.

- $u^T v$
- $v^T u$
- $\left(\frac{u^T u}{v^T u}\right) u$
- $\|u - v\|$

Problem 2 Determine if the following pair of vectors are orthogonal.

$$\left\{ \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2.5 \end{bmatrix} \right\}, \left\{ \begin{bmatrix} -3 \\ 7 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -8 \\ 25 \\ -7 \end{bmatrix} \right\}, \left\{ \begin{bmatrix} 13 \\ -3 \\ -7 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

Problem 3 Let $W = \text{colsp}(A)$, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 1 \end{bmatrix}.$$

Find W^\perp , the orthogonal complement of W .

Problem 4 Let

$$u = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix},$$

and $W = \text{span}\{u\}$. What is the dimension of W^\perp , the orthogonal complement of W .

Problem 5 Let A be a 7×5 matrix. What is the smallest possible dimension of $[\text{colsp}(A)]^\perp$?

Problem 6 Let

$$u_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \text{ and } x = \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}.$$

Express x as a linear combination of $\{u_1, u_2, u_3\}$.

Problem 7 Let

$$W = \text{span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\} \text{ and } \vec{y} = \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix}$$

Find $\text{proj}_W \vec{y}$.