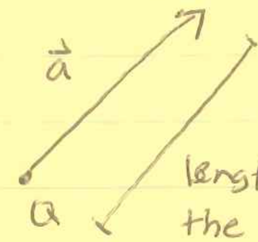
Lecture 1 Vectors and complex numbers

Many familiar physical notions such as forces, velocities, and accelerations, involve both a magnitude (amount of force, velocity or acceleration) and a direction

Vectors are represented as arrows



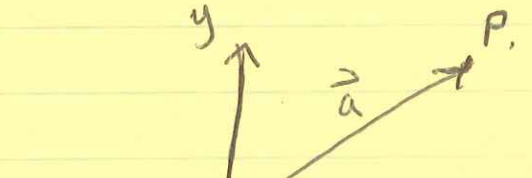
Vector acts at point Q

length represents the magnitude

direction of arrow ≡ direction of vector

In this class we will denote vectors by lower case letter with a superposed arrow (like $\vec{a}$)

It is common to use a reference coordinate system to define vectors. Consider the following coordinate system in 2-D space w.r.t. pt. Q

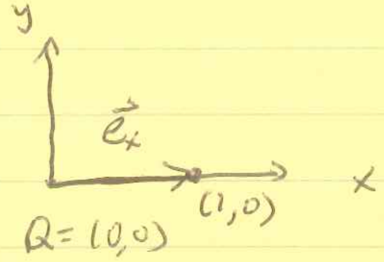


Assume $\vec{a}$ has endpoint $P = (a, b)$

Let Q be the origin $(0,0)$, then the vector is completely determined by its end point

We can define a unit vector by its end point (1, 0) along the x-axis as

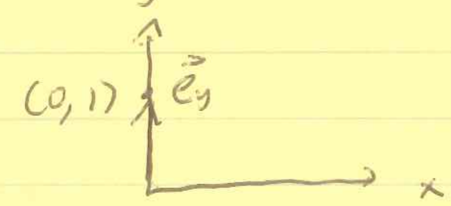
$$\vec{e}_x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$



Note: unit vector has length / magnitude of 1

Similarly, we define a unit vector along the y-axis as

$$\vec{e}_y = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



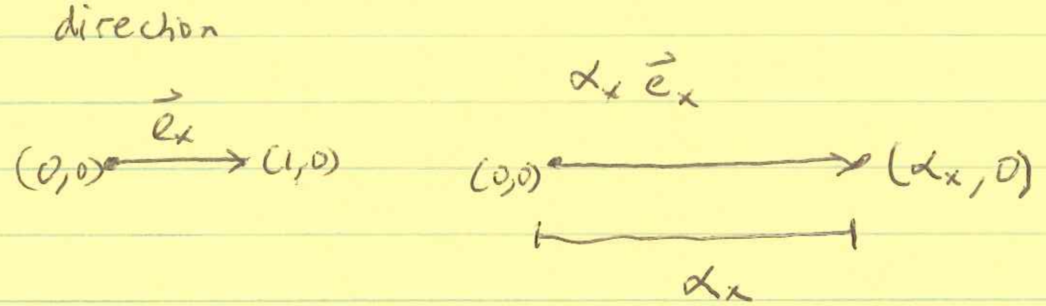
with endpoint (0, 1)

Then $\vec{a}$ can be represented as a linear combination of the two, that is

$$\vec{a} = \alpha_x \vec{e}_x + \alpha_y \vec{e}_y$$

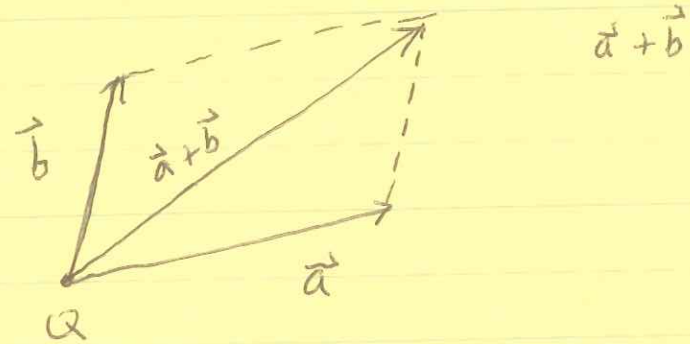
Note: lower case letters are scalars

Note that multiplication by scalar $\alpha_x \vec{e}_x$ affects only the length of the vector but not the direction

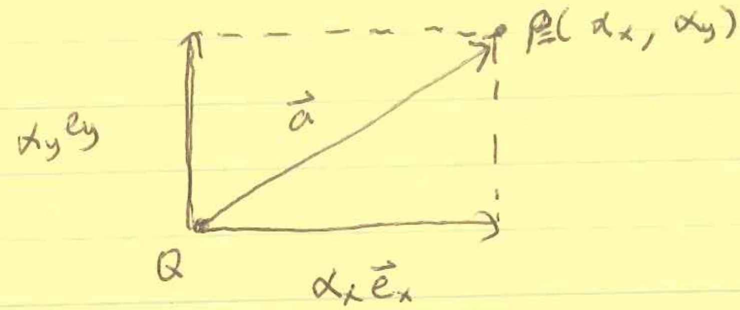


What does it mean to add vectors?

Parallelogram law for vector addition: The sum of two vectors $\vec{a}$ and $\vec{b}$ that act at the same point Q is the vector in the same point Q ~~par~~ parallelogram having $\vec{a}$ and $\vec{b}$ as adjacent sides that is represented by a diagonal beginning at Q



So what is $\vec{a} = \alpha_x \vec{e}_x + \alpha_y \vec{e}_y$?

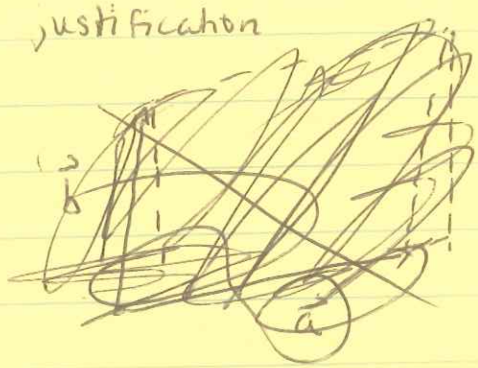


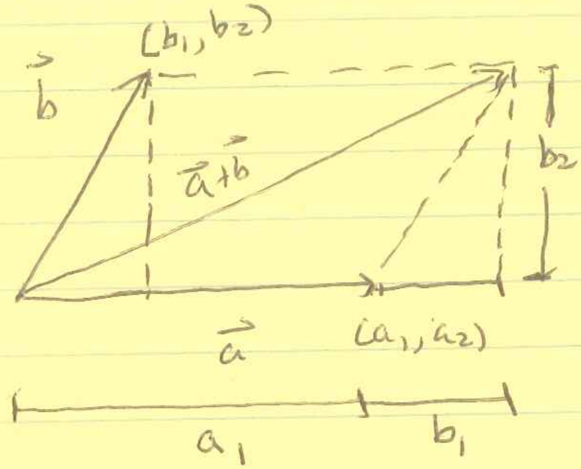
By the parallelogram law of addition of vectors, it follows that summing two vectors $\vec{a}+\vec{b}$ is a componentwise operation

Ex 1: $\alpha_x \vec{e}_x + \alpha_y \vec{e}_y = \begin{bmatrix} \alpha_x \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha_y \end{bmatrix} = \begin{bmatrix} \alpha_x \\ \alpha_y \end{bmatrix}$

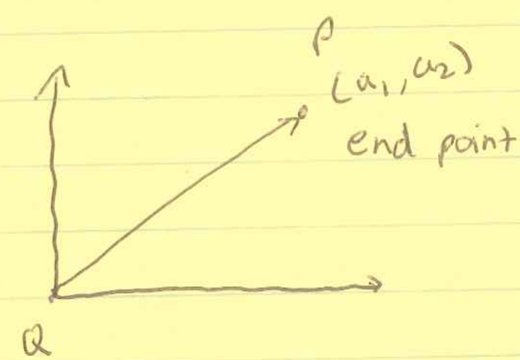
Ex 2: $\vec{a} + \vec{b} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} a_1+b_1 \\ a_2+b_2 \end{bmatrix}$

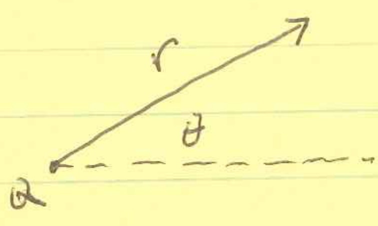
Visual justification





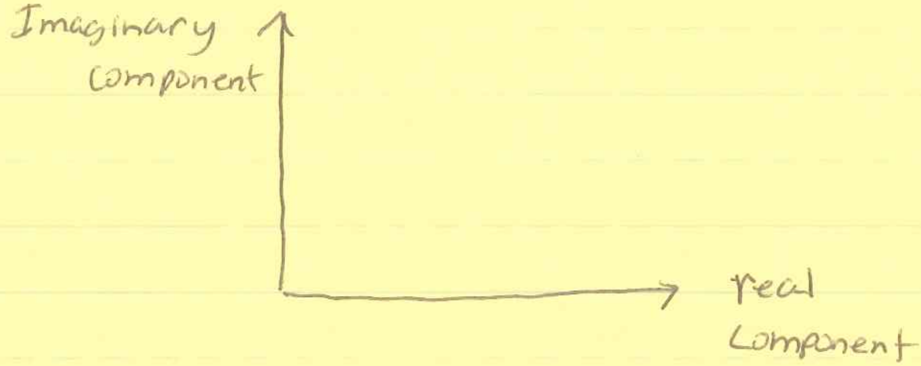
Note we can use polar coordinates to define (r, θ)
vector





(r, θ)
equally defines
your vector

Thus far, we have considered real values.
Consider a complex reference coordinate system



What is an imaginary component?
It is the scalar multiplying the quantity called "i" where $i \equiv \sqrt{-1}$ s.t. $i^2 = -1$

Imaginary numbers do exist and can be used to arrive at real solutions pertinent to the real word, eventhough they make no physical sense.

Ex. Quadratic formula

$$ax^2 + bx + c = 0$$

solution

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

case: $b^2 < 4ac \Rightarrow$ gives imaginary component

$$\frac{-b \pm i\sqrt{4ac - b^2}}{2a}$$