- $\det(A) = \lambda_1, \lambda_2, \ldots, \lambda_n$
- Computing orthogonal complement of a vector subspace
- Computing dimension of an orthogonal complement of a subspace
- Computing distance between two vectors
- Computing inner dot product for two vectors
- Computing projection of vector onto a subspace
- Finding coefficients in expansion of a vector as a linear combination of an orthogonal basis
- Determine orthogonality of a set of vectors

Additional Review Problems

Determine whether the following statements are true or false

1. Two row equivalent matrices have the same rank true
2. There exists a 3×2 matrix with rank 3. false $A = 3$ rank$(A) \le 2$
3. An homogeneous liner equation always has a solution true ($Ax = 0$, $x = 0$)
4. If a 3×3 matrix A has a zero row, then rank $A = 2$. False
5. If $v \in \mathbb{R}^n$, then $-v$ is in the $span\{v\}$ true
6. Let $u \in \mathbb{R}^n$ and $v \in \mathbb{R}^n$; then $span\{u, \underbrace{u - v}_{w}\}$ contains v. true $v = -w + u$
7. If a 6×4 matrix A has linearly independent columns, then the echelon form of A contains two zero rows. true
8. There exist a 3×5 matrix whose column space has dimension 4. False
9. If a square matrix A has two identical columns, then $\det(A) = 0$. true
10. If $\lambda = 0$ is an eigenvalue of the square matrix A, then A is invertible. False
11. Every invertible matrix is diagonalizable. False
12. Every diagonalizable matrix is invertible. False
13. The set of all solutions of a system of homogeneous equation with m equations and n unknowns is a subspace in $\mathbb{R}^m$. False $Ax = 0$
14. The set of all linear combinations of columns of an $m \times n$ matrix is a subspace in $\mathbb{R}^n$ False
15. The columns of an $n \times n$ matrix A form a basis for $colsp(A)$. False
16. The columns of an $n \times n$ invertible matrix from a basis for $\mathbb{R}^n$. true
17. If matrix A is row equivalent to matrix B, then $Ker(A) = Ker(B)$. true
18. Two similar matrices have the same eigenvectors. False

they have the same eigenvalues

$\det(A) = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdots \lambda_n$
if $\det(A) = 0$
A^{-1} does not exist
A is not full rank
A has linearly dependent columns/rows

basis $\Rightarrow$ linearly indep. set of vectors

A invertible
$\Rightarrow$ A has linearly indep. columns

$\dim(\mathbb{R}^n) = n$
$\dim(colsp(A)) = n$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad \frac{1}{1}$$

$$A = \begin{bmatrix} -1 & 2 \\ 0 & 5 \end{bmatrix} \qquad \lambda_1 = -1 \quad \lambda_2 = 5$$

diagonalizable

$$(A - \lambda I)\vec{v} = 0$$

$$\begin{bmatrix} -1 & 2 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 6 \end{bmatrix}$$

$$\begin{aligned} 2x_2 &= 0 \\ 6x_2 &= 0 \end{aligned} \Rightarrow x_2 = 0$$

x_1 free variable

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\lambda_2 = 5$

$$\begin{bmatrix} -1 & 2 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} -6 & 2 \\ 0 & 0 \end{bmatrix}$$

$$-6x_1 + 2x_2 = 0$$

$$2x_2 = 6x_1$$

$$x_2 = 3x_1$$

$$v_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$$

$\quad \bar{v}_1 \quad \bar{v}_2$

$$P^{-1} = \frac{1}{\det(P)} \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix} \qquad \det(P) = 3 - 0 = 3$$

$$P^{-1} = \begin{bmatrix} 1 & -1/3 \\ 0 & 1/3 \end{bmatrix}$$

$$D = P^{-1} A P = \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix}$$

$-P.$

$$A = PDP^{-1}$$
$$A^3 = PD^3P^{-1}$$

$$(PD\underbrace{P^{-1})(P}_{I}DP^{-1}) = A^2$$

$$(PD^2\underbrace{P^{-1})(P}_{I}DP^{-1}) = A^3$$

$$PD^3P^{-1}$$

$$D^3 = \begin{bmatrix} (-1)^3 & 0 \\ 0 & 5^3 \end{bmatrix}$$

$$\gamma_1 = -1 \quad \gamma_2 = 5^3$$

5) $\lambda = -5, \underbrace{1, 1}_{\text{algebraic multiplicity two!}}, 3$

If $\dim(E_{\lambda=1}) = 2$ diagonalizable
geometric multiplicity two

eigenspace for $\lambda = 1$

If " $= 1$ not diagonalizable
geometric multiplicity is 1

$\underbrace{(A - \lambda I)}V = 0$

$\lambda = 1$

$$\begin{bmatrix} -6 & 2 & -1 & 3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \boxed{x_4 = 0}$

$-6x_1 + 2x_2 - x_3 = 0$

$$\boxed{x_1 = \frac{2}{6}x_2 - \frac{1}{6}x_3}$$

let $x_3 = 1$ $x_2 = 1$ $x_2 \neq x_3$

x_1 $x_2 = x_3$

$$= \begin{bmatrix} 1/6 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$V = \begin{bmatrix} 2/6\, x_2 \\ x_2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1/6\, x_3 \\ 0 \\ x_3 \\ \ \end{bmatrix}.$$

$$V = x_2 \begin{bmatrix} 2/6 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1/6 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$E_{\lambda=1} = \text{span}\left\{ \begin{bmatrix} 2/6 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1/6 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$