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Lecture 5

From last Lecture THQuestion $\Rightarrow$

$$\begin{aligned} 5x_1 + 2x_2 + x_3 + 2x_4 &= 1 \\ 0\cdot x_1 + 0\cdot x_2 + 0\cdot x_3 + 2x_4 &= 5 \\ 0\cdot x_1 + 0\cdot x_2 + x_3 + 7x_4 &= 2 \end{aligned}$$

[E1] $L_i \leftrightarrow L_j$

[E2] $kL_i \rightarrow L_i$

[E3] $kL_j + L_i \rightarrow L_i$

$$\left(\begin{array}{c:c:cc:c} 5 & 2 & 1 & 2 & 1 \\ 0 & 0 & 0 & 2 & 5 \\ 0 & 0 & 1 & 7 & 2 \end{array}\right) \begin{matrix} \\ \updownarrow [E1] \end{matrix}$$

pivot $\rightarrow$ — free variable

$$\left(\begin{array}{cccc:c} \textcircled{5} & \boxed{2} & 1 & 2 & 1 \\ 0 & 0 & \textcircled{1} & 7 & 2 \\ 0 & 0 & 0 & \textcircled{2} & 5 \end{array}\right)$$

THQ1

$$\begin{aligned} x_1 + 2x_2 &= 0 \\ x_1 + 2x_2 &= 0 \end{aligned}$$

After $2L_1 \rightarrow L_1$

$$\Rightarrow \begin{aligned} x_1 + 2x_2 &= 0 \\ 0 + 0 &= 0 \end{aligned}$$

$-L_1 + L_2 \rightarrow L_2$

THQ2

$$\left(\begin{array}{ccc:c} \textcircled{1} & 5 & 2 & 3 \\ 0 & \textcircled{-9} & -3 & -5 \\ 0 & 0 & \textcircled{14/3} & \frac{-5}{9} + 5 \end{array}\right)$$

R3 $\quad x_3 = \left(\frac{-5}{9} + 5\right)\frac{3}{14}$

R2 $\quad -9x_2 - 3x_3 = -5$

$$x_2 = \frac{-5 + x_3}{-9}$$

R3 $\quad x_1 = f(x_3, x_2)$

Example
$$\begin{aligned} x_2 + x_3 &= 6 \\ x_1 + 4x_2 + 3x_3 &= 17 \\ x_1 + 2x_2 + x_3 &= 5 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & 6 \\ 1 & 4 & 3 & 17 \\ 1 & 2 & 1 & 5 \end{array}\right)$$

[E1]
$L_1 \Leftrightarrow L_2$
$$\left(\begin{array}{ccc|c} 1 & 4 & 3 & 17 \\ 0 & 1 & 1 & 6 \\ 1 & 2 & 1 & 5 \end{array}\right)$$

$-L_1 + L_3 \rightarrow L_3$
$$\left(\begin{array}{ccc|c} 1 & 4 & 3 & 17 \\ 0 & 1 & 1 & 6 \\ \cancel{0} & \cancel{-2} & \cancel{-2} & \cancel{-12} \end{array}\right)$$

delete, this is a multiple of L_2

~~pioo~~

pivot variables: x_1, x_2

free variable: x_3

$$\begin{aligned} x_1 + 4x_2 + 3x_3 &= 17 \\ x_2 + x_3 &= 6 \end{aligned}$$

Free to choose x_3, then solve for x_1, x_2

$$x_2 = 6 - x_3$$
$$x_1 = 17 - 4x_2 - 3x_3$$

$$-5x_5 = 0$$
$$x_5 = \frac{0}{-5} = 0$$

The other way $0 \cdot x_5 = -5$

$$0 \neq -5$$

Reduced Row Echelon form (unique representation)
(Row canonical form)

<u>Goal:</u> Make the pivot point equal to one, remove all possible off-diagonal coefficients (non-pivot points)

Example from lecture 4

$$\left(\begin{array}{cccc|c} 4 & 2 & 0 & 1 & 6 \\ 1 & 2 & 1 & 2 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 1 & 0 & 3 & 2 & 4 \end{array}\right)$$

⇒ After elementary operations
Echelon form

$$\left(\begin{array}{cccc|c} 4 & 2 & 0 & 1 & 6 \\ 0 & 3/2 & 1 & 7/4 & -1/2 \\ 0 & 0 & -2/3 & -1/6 & 7/3 \\ 0 & 0 & 0 & 18/12 & 14 \end{array}\right)$$

1st step make pivot points equal 1

$$\frac{1}{4}L_1 \to L_1$$

$$\frac{2}{3}L_2 \to L_2$$

$$-\frac{3}{2}L_3 \to L_3$$

$$\frac{2}{3}L_4 \to L_4$$

$$\left(\begin{array}{cccc|c} 1 & 1/2 & 0 & 1/4 & 6/4 \\ 0 & 1 & 2/3 & 7/6 & -1/3 \\ 0 & 0 & 1 & 1/4 & -7/2 \\ 0 & 0 & 0 & 1 & 28/3 \end{array}\right)$$

$$L_4\left(-\frac{1}{4}\right) + L_3 \to L_3$$

$$L_4\left(-\frac{7}{6}\right) + L_2 \to L_2$$

$$L_4\left(-\frac{1}{4}\right) + L_1 \to L_1$$

$$\left(\begin{array}{cccc|c} 1 & 1/2 & 0 & 0 & -1/4 \cdot \frac{28}{3} + 6/4 \\ & 1 & 2/3 & 0 & -7 \cdot 14/9 - 1/3 \\ & & 1 & 0 & -7/3 - 7/2 \\ & & & \underbrace{1} & \underbrace{28/3} \end{array}\right)$$

$x_4 = 28/3$

$X_3 = -7/3 - 7/2$

Keep going!

$$-\frac{2}{3}L_3 + L_2 \to L_2$$

$$-\frac{1}{2}L_2 + L_1 \to L_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{1}{2}\left(-\frac{84}{9} + 6/3\right) - \frac{28}{12} + 6/4 \\ & 1 & 0 & 0 & -84/9 + 6/3 \\ & & 1 & 0 & -7/3 - 7/2 \\ & & & 1 & 28/3 \end{array}\right]$$