AMS10 Midterm Version A

Instructions: No calculators or electronic devices allowed. A maximum of one page of notes (front and back) allowed. IMPORTANT: Please use a #2 pencil and mark the correct test version on the scantron. Answer keys are different!

Table 1: Trig table

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$-\pi/6$	$-\pi/4$	$-\pi/3$
arctan()	0	$1/\sqrt{3}$	1	$\sqrt{3}$	$-1/\sqrt{3}$	-1	$-\sqrt{3}$

Question 1.

If $A = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$, what is the product AA^T, where $A^T = \begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix}$ is the transpose?

a) $AA^T = \begin{bmatrix} 5 & 2 \\ 2 & 4 \end{bmatrix}$

b) $AA^T = \begin{bmatrix} -3 & 2 \\ 2 & 4 \end{bmatrix}$

c) $AA^T = \begin{bmatrix} -3 & 2 \\ 2 & 0 \end{bmatrix}$

d) $AA^T = \begin{bmatrix} -3 & 2 \\ -4 & 4 \end{bmatrix}$

Question 2.

If $A = \begin{bmatrix} -10 & 0 \\ 0 & 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$, what is $A\vec{v}$?

a) $A\vec{v} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

b) $A\vec{v} = \begin{bmatrix} -50 \\ 4 \end{bmatrix}$

c) $A\vec{v} = -46$

d) $A\vec{v} = 54$

Question 3.

If $A \in \mathbb{R}^{n\times m}$ and $B \in \mathbb{R}^{n\times p}$, then

a) $AB \in \mathbb{R}^{n\times p}$

b) AB Does not exist

c) $AB \in \mathbb{R}^{m\times p}$

d) $AB \in \mathbb{R}^{m\times n}$

Question 4.

Select all the matrices that are in echelon form?

a) $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 & 0 & -10 & -9 \\ 0 & 0 & 1 & -7 & -7 \\ 0 & 0 & 2 & -14 & -14 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Question 5.

Consider a linear system $A\vec{x} = \vec{b}$ with 4 unknown variables $\vec{x} = [x_1, x_2, x_3, x_4]^T$. The augmented matrix $M = [A|\vec{b}]$ has the reduced matrix $\left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$, what are the pivot **variables**?

a) x_1 b) x_3, x_2 c) 1,2

d) x_1, x_4 e) x_1, x_2, x_3

Question 6.

Consider a linear system $A\vec{x} = \vec{b}$ with 4 unknown variables $\vec{x} = [x_1, x_2, x_3, x_4]^T$. The augmented matrix $M = [A|\vec{b}]$ has the reduced matrix $\left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$, what are the free **variables**?

a) x_1 b) x_3, x_2 c) 1,2

d) x_1, x_4 e) x_1, x_2, x_3

Question 7.

Select all the matrices that are in row canonical form?

a) $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & -2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 1 & -10 & -9 \\ 0 & 1 & 1 & -7 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Question 8.

How many solutions does $x^{20} = 1$ have including complex solutions?

a) 20 b) 19 c) 1 d) 2

Question 9.

Given the complex number $z = \sqrt{2} - i\sqrt{2}$, the absolute value of z is

a) $2\sqrt{2}$ b) $\sqrt{2}$ c) 2

d) 4

Question 10.

Given the complex number $z = \sqrt{2} - i\sqrt{2}$, the argument of z is

a) $\arctan(1)$ b) $\arctan(-1)$ c) $\arctan(-1) + \pi$

d) $\arctan(1) + \pi$

Question 11.
Given the complex number $z = \sqrt{2} - i\sqrt{2}$, find the real and imaginary part of z^4

a) $Re(z) = 2^4$ and $Im(z) = 2^4$

b) $Re(z) = -2^4$ and $Im(z) = -2^4$

c) $Re(z) = 2^4$ and $Im(z) = 0$

d) $Re(z) = -2^4$ and $Im(z) = 0$

Question 12.
Given $A^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ solve $A\vec{x} = \vec{b}$ for $\vec{x}$, where $\vec{b} = [-2, 1]^T$. Which of the following is a solution?

a) $\vec{x} = \begin{bmatrix} -4 \\ 1/2 \end{bmatrix}$ b) $\vec{x} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$ c) no solution d) infinite solutions

Question 13.
How many solutions does $A\vec{x} = \vec{b}$ have? The augmented matrix is $[A|b] = \left[\begin{array}{cccc} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 3 \end{array}\right]$.

a) no solution

b) infinite solutions

c) insufficient information

d) one solution

Question 14.
Given $[A|b] = \left[\begin{array}{cccc} -1 & 8 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 6 & 3 \end{array}\right]$ how many solutions does $A\vec{x} = \vec{b}$ have?

a) no solution

b) infinite solutions

c) insufficient information

d) one solution

Question 15.
Given $[A|b] = \left[\begin{array}{cccc} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 1 & 5 & 10 \end{array}\right]$ how many solutions does $A\vec{x} = \vec{b}$ have?

a) no solution

b) infinite solutions

c) insufficient information

d) one solution

Question 16.
Which of the following matrices can NOT be arrived at through a series of elementary operations on $A_1 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (i.e. is NOT row equivalent)? Hint: Apply Gaussian elimination techniques.

a) $A_2 = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

b) $A_2 = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$

c) $A_2 = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

d) $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Question 17.

What is the inverse of $A = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$?

a) $A^{-1} = \begin{bmatrix} -2 & -2 \\ 0 & 1 \end{bmatrix}$

b) $A^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1/2 \end{bmatrix}$

c) $A^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & -1/2 \end{bmatrix}$

d) $A^{-1} = \begin{bmatrix} 1 & 1/2 \\ 0 & -1/2 \end{bmatrix}$

Question 18.
Select all the sets of vectors that are linearly independent?

a) $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, 2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} \right\}$

Question 19.
Which of the following matrices has $\det(A) = 0$? Select all that apply.

a) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & -6 \\ 1 & 1 \end{bmatrix}$

c) . $A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

d) $A = -8$

Question 20.

Given a matrix: $B = \begin{bmatrix} 1 & 0 & -2 \\ 2 & \beta & 2\beta \\ -4 & 1 & 0 \end{bmatrix}$. Which value of β makes B non-invertible?

a) $-\frac{1}{10}$

b) $-\frac{1}{2}$

c) $-\frac{2}{5}$

d) $-\frac{1}{5}$

Question 21.

Vector $\vec{v} = [-1, 1]^T$ is a **unique** linear combination of what set? Select all that apply.

a) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}$

Question 22.

Vector $\vec{v} = [5, 8]^T$ is in the span of what set? Select all that apply.

a) $\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

Question 23.

What is $\dim(rowsp(A))$, where $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$?

a) 1

b) 2

c) 3

d) 4

Question 24.

What is $\dim(colsp(A))$, where $A = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 8 & 1 & 0 & 6 \\ 3 & 0 & 2 & 2 \end{bmatrix}$?

a) 1

b) 2

c) 3

d) 4

Question 25.

Select all matrices below that have $dim(colsp(A)) = 2$.

a) $A = \begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ c) $A = \begin{bmatrix} 1 & 0 & 2 & -3 & 1 \\ 2 & 0 & 4 & -6 & 2 \end{bmatrix}$

Question 26.

Select all matrices below that have $colsp(A) = \mathbb{R}^2$?

a) $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & -1 \\ -3 & 3 \end{bmatrix}$

c) $A = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ d) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

Question 27.

What is the rank of matrix $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$?

a) 1 b) 2

c) 3 d) 4

Question 28.

Select all sets that serve as a basis for the span$\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right)$.

a) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$ b) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^T \right\}$ d) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

Question 29.

Select all sets that serve as a basis for $colsp(A)$, where $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 0 \\ 0 & -1 & -5 & 0 \end{bmatrix}$?

a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 5 \\ 0 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

Question 30.

Given the rank-nullity theorem, what is the $\dim(Ker(A))$ if $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$?

a) 0 b) 1 c) 2 d) 3 e) 4

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