

## AMS10 Midterm Version B

Instructions: No calculators or electronic devices allowed. A maximum of one page of notes (front and back) allowed. IMPORTANT: Please use a #2 pencil and mark the correct test version on the scantron. Answer keys are different!

Table 1: Trig table

$\theta$	0	$\pi/6$	$\pi/4$	$\pi/3$	$-\pi/6$	$-\pi/4$	$-\pi/3$
$\arctan()$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	$-1/\sqrt{3}$	-1	$-\sqrt{3}$

### Question 1.

How many solutions does  $x^{20} = 1$  have including complex solutions?

- a) 20                                      b) 19                                      c) 1                                      d) 2

### Question 2.

Given the complex number  $z = \sqrt{2} - i\sqrt{2}$ , the absolute value of  $z$  is

- a)  $2\sqrt{2}$                                       b)  $\sqrt{2}$                                       c) 2  
d) 4

### Question 3.

Given the complex number  $z = \sqrt{2} - i\sqrt{2}$ , the argument of  $z$  is

- a)  $\arctan(1)$                                       b)  $\arctan(-1)$                                       c)  $\arctan(-1) + \pi$   
d)  $\arctan(1) + \pi$

### Question 4.

Given the complex number  $z = \sqrt{2} - i\sqrt{2}$ , find the real and imaginary part of  $z^4$

- a)  $Re(z) = 2^4$  and  $Im(z) = 2^4$                                       b)  $Re(z) = -2^4$  and  $Im(z) = -2^4$   
c)  $Re(z) = 2^4$  and  $Im(z) = 0$                                       d)  $Re(z) = -2^4$  and  $Im(z) = 0$

### Question 5.

If  $A = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$ , what is the product  $AA^T$ , where  $A^T = \begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix}$  is the transpose?

- a)  $AA^T = \begin{bmatrix} 5 & 2 \\ 2 & 4 \end{bmatrix}$                                       b)  $AA^T = \begin{bmatrix} -3 & 2 \\ 2 & 4 \end{bmatrix}$   
c)  $AA^T = \begin{bmatrix} -3 & 2 \\ 2 & 0 \end{bmatrix}$                                       d)  $AA^T = \begin{bmatrix} -3 & 2 \\ -4 & 4 \end{bmatrix}$

**Question 6.**

If  $A = \begin{bmatrix} -10 & 0 \\ 0 & 1 \end{bmatrix}$  and  $\vec{v} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ , what is  $A\vec{v}$ ?

- a)  $A\vec{v} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$       b)  $A\vec{v} = \begin{bmatrix} -50 \\ 4 \end{bmatrix}$       c)  $A\vec{v} = -46$       d)  $A\vec{v} = 54$

**Question 7.**

If  $A \in \mathbb{R}^{n \times m}$  and  $B \in \mathbb{R}^{n \times p}$ , then

- a)  $AB \in \mathbb{R}^{n \times p}$       b)  $AB$  Does not exist      c)  $AB \in \mathbb{R}^{m \times p}$       d)  $AB \in \mathbb{R}^{m \times n}$

**Question 8.**

Select all the matrices that are in echelon form?

- a)  $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & -2 \end{bmatrix}$       b)  $\begin{bmatrix} 1 & 1 & 0 & -10 & -9 \\ 0 & 0 & 1 & -7 & -7 \\ 0 & 0 & 2 & -14 & -14 \end{bmatrix}$
- c)  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$       d)  $\begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

**Question 9.**

Consider a linear system  $A\vec{x} = \vec{b}$  with 4 unknown variables  $\vec{x} = [x_1, x_2, x_3, x_4]^T$ . The augmented matrix  $M = [A|\vec{b}]$  has the reduced matrix  $\begin{bmatrix} 1 & 2 & 1 & 1 & | & 1 \\ 0 & 0 & 0 & 2 & | & 1 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$ , what are the pivot **variables**?

- a)  $x_1$       b)  $x_3, x_2$       c) 1,2  
d)  $x_1, x_4$       e)  $x_1, x_2, x_3$

**Question 10.**

Consider a linear system  $A\vec{x} = \vec{b}$  with 4 unknown variables  $\vec{x} = [x_1, x_2, x_3, x_4]^T$ . The augmented matrix  $M = [A|\vec{b}]$  has the reduced matrix  $\begin{bmatrix} 1 & 2 & 1 & 1 & | & 1 \\ 0 & 0 & 0 & 2 & | & 1 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$ , what are the free **variables**?

- a)  $x_1$       b)  $x_3, x_2$       c) 1,2  
d)  $x_1, x_4$       e)  $x_1, x_2, x_3$

**Question 11.**

Select all the matrices that are in row canonical form?

a)  $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

b)  $\begin{bmatrix} 1 & 0 & 1 & -10 & -9 \\ 0 & 1 & 1 & -7 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

c)  $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

d)  $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

**Question 12.**

Which of the following matrices can NOT be arrived at through a series of elementary operations on  $A_1 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  (i.e. is NOT row equivalent)? Hint: Apply Gaussian elimination techniques.

a)  $A_2 = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

b)  $A_2 = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$

c)  $A_2 = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

d)  $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**Question 13.**

What is the inverse of  $A = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$ ?

a)  $A^{-1} = \begin{bmatrix} -2 & -2 \\ 0 & 1 \end{bmatrix}$

b)  $A^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1/2 \end{bmatrix}$

c)  $A^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & -1/2 \end{bmatrix}$

d)  $A^{-1} = \begin{bmatrix} 1 & 1/2 \\ 0 & -1/2 \end{bmatrix}$

**Question 14.**

Given  $A^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$  solve  $A\vec{x} = \vec{b}$  for  $\vec{x}$ , where  $\vec{b} = [-2, 1]^T$ . Which of the following is a solution?

a)  $\vec{x} = \begin{bmatrix} -4 \\ 1/2 \end{bmatrix}$

b)  $\vec{x} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

c) no solution

d) infinite solutions

**Question 15.**

How many solutions does  $A\vec{x} = \vec{b}$  have? The augmented matrix is  $[A|b] = \left[ \begin{array}{ccc|c} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 3 \end{array} \right]$ .

a) no solution

b) infinite solutions

c) insufficient information

d) one solution

**Question 16.**

Given  $[A|b] = \left[ \begin{array}{ccc|c} -1 & 8 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 6 & 3 \end{array} \right]$  how many solutions does  $A\vec{x} = \vec{b}$  have?

- a) no solution
- b) infinite solutions
- c) insufficient information
- d) one solution

**Question 17.**

Given  $[A|b] = \left[ \begin{array}{ccc|c} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 1 & 5 & 10 \end{array} \right]$  how many solutions does  $A\vec{x} = \vec{b}$  have?

- a) no solution
- b) infinite solutions
- c) insufficient information
- d) one solution

**Question 18.**

Vector  $\vec{v} = [-1, 1]^T$  is a **unique** linear combination of what set? Select all that apply.

- a)  $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- b)  $\left\{ \begin{bmatrix} 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$
- c)  $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \end{bmatrix} \right\}$
- d)  $\left\{ \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}$

**Question 19.**

Vector  $\vec{v} = [5, 8]^T$  is in the span of what set? Select all that apply.

- a)  $\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \end{bmatrix} \right\}$
- b)  $\left\{ \begin{bmatrix} 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- c)  $\left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$
- d)  $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

**Question 20.**

Select all matrices below that have  $\dim(\text{colsp}(A)) = 2$ .

- a)  $A = \begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
- b)  $A = \begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
- c)  $A = \begin{bmatrix} 1 & 0 & 2 & -3 & 1 \\ 2 & 0 & 4 & -6 & 2 \end{bmatrix}$

**Question 21.**

Select all matrices below that have  $\text{colsp}(A) = \mathbb{R}^2$ ?

a)  $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \end{bmatrix}$

b)  $A = \begin{bmatrix} 1 & -1 \\ -3 & 3 \end{bmatrix}$

c)  $A = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

d)  $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

**Question 22.**

What is the rank of matrix  $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ ?

a) 1

b) 2

c) 3

d) 4

**Question 23.**

Select all sets that serve as a basis for the span  $\left( \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right)$ .

a)  $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$

b)  $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

c)  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^T \right\}$

d)  $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

**Question 24.**

Select all sets that serve as a basis for  $\text{colsp}(A)$ , where  $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 0 \\ 0 & -1 & -5 & 0 \end{bmatrix}$ ?

a)  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

b)  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 5 \\ 0 \end{bmatrix} \right\}$

c)  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

d)  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$



**Question 30.**

Given a matrix:  $B = \begin{bmatrix} 1 & 0 & -2 \\ 2 & \beta & 2\beta \\ -4 & 1 & 0 \end{bmatrix}$ . Which value of  $\beta$  makes  $B$  non-invertible?

a)  $-\frac{1}{10}$

b)  $-\frac{1}{2}$

c)  $-\frac{2}{5}$

d)  $-\frac{1}{5}$

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