AMS10 Midterm: Version A

Instructions: No calculators or electronic devices allowed. A maximum of two pages of notes (front and back) allowed. IMPORTANT: Please use a #2 pencil and ParScore f-1712 scantron (red one). Do not begin the exam until instructed to do so. Please bubble in "A" under "Test Form."

Question 1.
How many complex solutions does $x^3 = 1$ have?

a) 1 b) 2 c) 0 d) 3

Question 2.
What is the complex exponential form of $z = -1 + 5i$?

a) $\sqrt{26}e^{i(\arctan(-5))}$ b) $\sqrt{26}e^{i(\arctan(-5)+\pi)}$ c) $\sqrt{24}e^{i(\arctan(-5)+\pi)}$

d) $\sqrt{26}e^{i(\arctan(-1/5))}$ e) $\sqrt{24}e^{i(\arctan(-1/5)+\pi)}$ f) $\sqrt{24}e^{i(\arctan(-5))}$

Question 3.
If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 \\ 2 & 5 \end{bmatrix}$, what is the product AB?

a) $AB = \begin{bmatrix} 2 & -4 \\ 2 & 6 \end{bmatrix}$ b) $AB = \begin{bmatrix} 1 & -4 \\ 2 & 5 \end{bmatrix}$

c) $AB = \begin{bmatrix} 1 & 0 & 1 & -4 \\ 0 & 1 & 2 & 5 \end{bmatrix}$ d) $AB = \begin{bmatrix} 1 & 2 \\ -4 & 5 \end{bmatrix}$

Question 4.
If $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, what is $A\vec{v}$?

a) $A\vec{v} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ b) $A\vec{v} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ c) $A\vec{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ d) $A\vec{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Question 5.
If $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{n\times p}$, then

a) $AB \in \mathbb{R}^{n\times p}$ b) Does not exist c) $AB \in \mathbb{R}^{m\times p}$ d) $AB \in \mathbb{R}^{m\times n}$

Question 6.
Which of the following matrices is in echelon form?

a) $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & -2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 & 0 & -10 & -9 \\ 0 & 0 & 1 & -7 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ d) All of the above

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Question 7.

Consider a linear system $A\vec{x} = \vec{b}$ with 4 unknown variables $\vec{x} = [x_1, x_2, x_3, x_4]^T$. The augmented matrix $M = [A|\vec{b}]$ has the reduced matrix $\begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 0 & 1 & 3 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, what are the pivot variables?

a) x_1 b) x_3, x_4 c) 1,1

d) x_1, x_2 e) x_2, x_3, x_4 f) $3, 2$

Question 8.

Consider a linear system $A\vec{x} = \vec{b}$ with 4 unknown variables $\vec{x} = [x_1, x_2, x_3, x_4]^T$. The augmented matrix $M = [A|\vec{b}]$ has the reduced matrix $\begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 0 & 1 & 3 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, what are the free variables?

a) x_1 b) x_3, x_4 c) 1,1

d) x_1, x_2 e) x_2, x_3, x_4 f) $3, 2$

Question 9.

Which of the following matrices is in row canonical form?

a) $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 & 0 & -10 & -9 \\ 0 & 0 & 1 & -7 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Question 10.

What is the inverse of $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$?

a) $A^{-1} = \begin{bmatrix} 1 & 1 & -1/4 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b) $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $A^{-1} = \begin{bmatrix} 1 & 1/2 & 1/4 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d) $A^{-1} = \begin{bmatrix} 1 & 1 & -4 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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Question 11.

What is the row canonical form of $A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 \end{bmatrix}$?

a) Already in row canonical form

b) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Question 12.

Given $A^{-1} = \begin{bmatrix} 1 & -2 \\ 1/4 & 1 \end{bmatrix}$ solve $A\vec{x} = \vec{b}$ for $\vec{b} = [-2, 1]^T$.

a) $\vec{x} = \begin{bmatrix} -4 \\ 1/2 \end{bmatrix}$

b) $\vec{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

c) no solution

d) infinite solutions

Question 13.

How many solutions does $A\vec{x} = \vec{b}$ have? The echelon form of the augmented matrix is $[A|b] = \begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 0 & 3 \end{bmatrix}$.

a) no solution

b) infinite solutions

c) insufficient information

d) one solution

Question 14.

Given $[A|b] = \begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 0 & 5 & 10 \\ 0 & -1 & 6 & 3 \end{bmatrix}$ how many solutions does $A\vec{x} = \vec{b}$ have?

a) no solution

b) infinite solutions

c) insufficient information

d) one solution

Question 15.

Given $[A|b] = \begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ how many solutions does $A\vec{x} = \vec{b}$ have?

a) no solution

b) infinite solutions

c) insufficient information

d) one solution

Question 16.
Which of the following sets of vectors are independent?

a) $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

Question 17.
Vector $\vec{v} = [1, 0]^T$ is a linear combination of what set?

a) $\left\{ \begin{bmatrix} 10 \\ 0 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

d) All of the above

Question 18.
Vector $\vec{v} = [-1, 1]^T$ is a **unique** linear combination of what set?

a) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \end{bmatrix} \right\}$

d) All of the above

Question 19.
Vector $\vec{v} = [3, 2]^T$ is in the span of what set?

a) $\left\{ \begin{bmatrix} 1 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

d) All of the above

Question 20.
Which matrix below has $colsp(A) \in \mathbb{R}^5$?

a) $\begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 2 & -3 & 1 \\ 0 & 1 & 5 & 10 & -1 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$

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Question 21.

Which matrix below has $colsp(A) = \mathbb{R}^3$?

a) $\begin{bmatrix} 1 & -2 \\ -3 & 1 \\ 5 & 6 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -1 & 2 \\ -3 & 1 & -6 \\ 5 & 0 & 10 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

d) None of the above

Question 22.

What is the rank of matrix $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 0 \\ 2 & 0 & 4 & 4 \end{bmatrix}$?

a) 1

b) 2

c) 3

d) 4

Question 23.

Which of the following sets is a basis for $rowsp(A)$, where $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \end{bmatrix}$?

a) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}^T \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}^T \right\}$

d) None of the above

Question 24.

Which of the following sets is a basis for $colsp(A)$, where $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 0 \end{bmatrix}$?

a) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \\ 5 \\ 0 \end{bmatrix}^T \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T, \begin{bmatrix} -1 \\ 1 \end{bmatrix}^T \right\}$

d) None of the above

Question 25.

What is $\dim(rowsp(A))$, where $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$?

a) 1 b) 2

c) 3 d) 4

Question 26.

What is $\dim(colsp(A))$, where $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 8 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 \\ 0 & 1 & 0 & 5 \end{bmatrix}$?

a) 1 b) 2

c) 3 d) 4

Question 27.

Given the rank-nullity theorem, what is the $\dim(Ker(A))$ if $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$?

a) 0 b) 1 c) 2 d) 3 e) 4