

## AMS10 Midterm: Version A

Instructions: No calculators or electronic devices allowed. A maximum of two pages of notes (front and back) allowed. IMPORTANT: Please use a #2 pencil and ParScore f-1712 scantron (red one). Do not begin the exam until instructed to do so. Please bubble in "A" under "Test Form."

**Question 1.**

How many complex solutions does  $x^3 = 1$  have?

- a) 1                                      b) 2                                      c) 0                                      d) 3

**Question 2.**

What is the complex exponential form of  $z = -1 + 5i$ ?

- a)  $\sqrt{26}e^{i(\arctan(-5))}$                       b)  $\sqrt{26}e^{i(\arctan(-5)+\pi)}$                       c)  $\sqrt{24}e^{i(\arctan(-5)+\pi)}$   
 d)  $\sqrt{26}e^{i(\arctan(-1/5))}$                       e)  $\sqrt{24}e^{i(\arctan(-1/5)+\pi)}$                       f)  $\sqrt{24}e^{i(\arctan(-5))}$

**Question 3.**

If  $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & -4 \\ 2 & 5 \end{bmatrix}$ , what is the product  $AB$ ?

- a)  $AB = \begin{bmatrix} 2 & -4 \\ 2 & 6 \end{bmatrix}$                                       b)  $AB = \begin{bmatrix} 1 & -4 \\ 2 & 5 \end{bmatrix}$   
 c)  $AB = \begin{bmatrix} 1 & 0 & 1 & -4 \\ 0 & 1 & 2 & 5 \end{bmatrix}$                                       d)  $AB = \begin{bmatrix} 1 & 2 \\ -4 & 5 \end{bmatrix}$

**Question 4.**

If  $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$  and  $\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ , what is  $A\vec{v}$ ?

- a)  $A\vec{v} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$                       b)  $A\vec{v} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$                       c)  $A\vec{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$                       d)  $A\vec{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

**Question 5.**

If  $A \in \mathbb{R}^{m \times n}$  and  $B \in \mathbb{R}^{n \times p}$ , then

- a)  $AB \in \mathbb{R}^{n \times p}$                       b) Does not exist                      c)  $AB \in \mathbb{R}^{m \times p}$                       d)  $AB \in \mathbb{R}^{m \times n}$

**Question 6.**

Which of the following matrices is in echelon form?

- a)  $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & -2 \end{bmatrix}$                                       b)  $\begin{bmatrix} 1 & 1 & 0 & -10 & -9 \\ 0 & 0 & 1 & -7 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$   
 c)  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$                                       d) All of the above

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**Question 7.**

Consider a linear system  $A\vec{x} = \vec{b}$  with 4 unknown variables  $\vec{x} = [x_1, x_2, x_3, x_4]^T$ . The augmented matrix  $M = [A|\vec{b}]$  has the reduced matrix  $\begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 0 & 1 & 3 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ , what are the pivot variables?

- a)  $x_1$     b)  $x_3, x_4$     c) 1,1  
d)  $x_1, x_2$     e)  $x_2, x_3, x_4$     f) 3,2

**Question 8.**

Consider a linear system  $A\vec{x} = \vec{b}$  with 4 unknown variables  $\vec{x} = [x_1, x_2, x_3, x_4]^T$ . The augmented matrix  $M = [A|\vec{b}]$  has the reduced matrix  $\begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 0 & 1 & 3 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ , what are the free variables?

- a)  $x_1$     b)  $x_3, x_4$     c) 1,1  
d)  $x_1, x_2$     e)  $x_2, x_3, x_4$     f) 3,2

**Question 9.**

Which of the following matrices is in row canonical form?

- a)  $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & -2 \end{bmatrix}$     b)  $\begin{bmatrix} 1 & 1 & 0 & -10 & -9 \\ 0 & 0 & 1 & -7 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$   
c)  $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$     d)  $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

**Question 10.**

What is the inverse of  $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ?

- a)  $A^{-1} = \begin{bmatrix} 1 & 1 & -1/4 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$     b)  $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$   
c)  $A^{-1} = \begin{bmatrix} 1 & 1/2 & 1/4 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$     d)  $A^{-1} = \begin{bmatrix} 1 & 1 & -4 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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**Question 11.**

What is the row canonical form of  $A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 \end{bmatrix}$ ?

- a) Already in row canonical form
- b)  $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
- c)  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
- d)  $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

**Question 12.**

Given  $A^{-1} = \begin{bmatrix} 1 & -2 \\ 1/4 & 1 \end{bmatrix}$  solve  $A\vec{x} = \vec{b}$  for  $\vec{b} = [-2, 1]^T$ .

- a)  $\vec{x} = \begin{bmatrix} -4 \\ 1/2 \end{bmatrix}$
- b)  $\vec{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- c) no solution
- d) infinite solutions

**Question 13.**

How many solutions does  $A\vec{x} = \vec{b}$  have? The echelon form of the augmented matrix is  $[A|\vec{b}] = \begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ .

- a) no solution
- b) infinite solutions
- c) insufficient information
- d) one solution

**Question 14.**

Given  $[A|\vec{b}] = \begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 0 & 5 & 10 \\ 0 & -1 & 6 & 3 \end{bmatrix}$  how many solutions does  $A\vec{x} = \vec{b}$  have?

- a) no solution
- b) infinite solutions
- c) insufficient information
- d) one solution

**Question 15.**

Given  $[A|\vec{b}] = \begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$  how many solutions does  $A\vec{x} = \vec{b}$  have?

- a) no solution
- b) infinite solutions
- c) insufficient information
- d) one solution

**Question 16.**

Which of the following sets of vectors are independent?

a)  $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

b)  $\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \end{bmatrix} \right\}$

c)  $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$

d)  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

**Question 17.**

Vector  $\vec{v} = [1, 0]^T$  is a linear combination of what set?

a)  $\left\{ \begin{bmatrix} 10 \\ 0 \end{bmatrix} \right\}$

b)  $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

c)  $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

d) All of the above

**Question 18.**

Vector  $\vec{v} = [-1, 1]^T$  is a **unique** linear combination of what set?

a)  $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

b)  $\left\{ \begin{bmatrix} 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

c)  $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \end{bmatrix} \right\}$

d) All of the above

**Question 19.**

Vector  $\vec{v} = [3, 2]^T$  is in the span of what set?

a)  $\left\{ \begin{bmatrix} 1 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$

b)  $\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

c)  $\left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

d) All of the above

**Question 20.**

Which matrix below has  $\text{colsp}(A) \in \mathbb{R}^5$ ?

a)  $\begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b)  $\begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

c)  $\begin{bmatrix} 1 & 0 & 2 & -3 & 1 \\ 0 & 1 & 5 & 10 & -1 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$

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**Question 21.**

Which matrix below has  $\text{colsp}(A) = \mathbb{R}^3$ ?

a)  $\begin{bmatrix} 1 & -2 \\ -3 & 1 \\ 5 & 6 \end{bmatrix}$

b)  $\begin{bmatrix} 1 & -1 & 2 \\ -3 & 1 & -6 \\ 5 & 0 & 10 \end{bmatrix}$

c)  $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

d) None of the above

**Question 22.**

What is the rank of matrix  $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 0 \\ 2 & 0 & 4 & 4 \end{bmatrix}$ ?

a) 1

b) 2

c) 3

d) 4

**Question 23.**

Which of the following sets is a basis for  $\text{rowsp}(A)$ , where  $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \end{bmatrix}$ ?

a)  $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$

b)  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}^T \right\}$

c)  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}^T \right\}$

d) None of the above

**Question 24.**

Which of the following sets is a basis for  $\text{colsp}(A)$ , where  $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 0 \end{bmatrix}$ ?

a)  $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

b)  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \\ 5 \\ 0 \end{bmatrix}^T \right\}$

c)  $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T, \begin{bmatrix} -1 \\ 1 \end{bmatrix}^T \right\}$

d) None of the above

**Question 25.**

What is  $\dim(\text{rowsp}(A))$ , where  $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$ ?

- a) 1
- b) 2
- c) 3
- d) 4

**Question 26.**

What is  $\dim(\text{colsp}(A))$ , where  $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 8 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 \\ 0 & 1 & 0 & 5 \end{bmatrix}$ ?

- a) 1
- b) 2
- c) 3
- d) 4

**Question 27.**

Given the rank-nullity theorem, what is the  $\dim(\text{Ker}(A))$  if  $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ ?

- a) 0
- b) 1
- c) 2
- d) 3
- e) 4